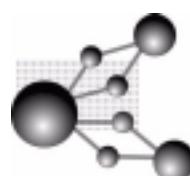


## André-Marie Tremblay



CENTRE DE RECHERCHE SUR LES PROPRIÉTÉS  
ÉLECTRONIQUES  
DE MATÉRIAUX AVANCÉS



## Sponsors:



# Why is the high- $T_c$ problem so difficult?

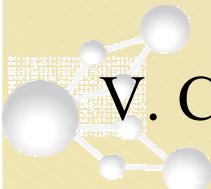
I. Standard paradigm

II. Why is there a problem with standard approaches?

III. Microscopic model

IV. Theoretical difficulties

- (a) Straight numerical approaches
- (b) Inadequacies of mean-field theory in low dimension
- (c) Approaching from weak coupling
- (d) Approaching from strong coupling
- (e) Phonons
- (f) Inhomogeneities



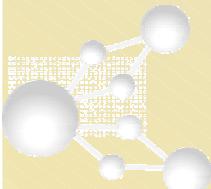
V. Conclusion

# I. Standard Paradigm

Theory of solids

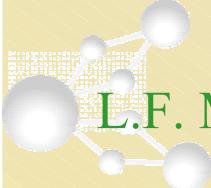
$$H = \text{Kinetic} + \text{Coulomb}$$

- Many new ideas and concepts needed for progress  
(Born-Oppenheimer, H-F, Bands...)
- Successful program
  - Semiconductors, metals *and superconductors*
  - Magnets
- Is there anything left to do?
  - Unexplained materials: High T<sub>c</sub>, Organics...
  - Strong correlations:  
strong interactions, low dimension

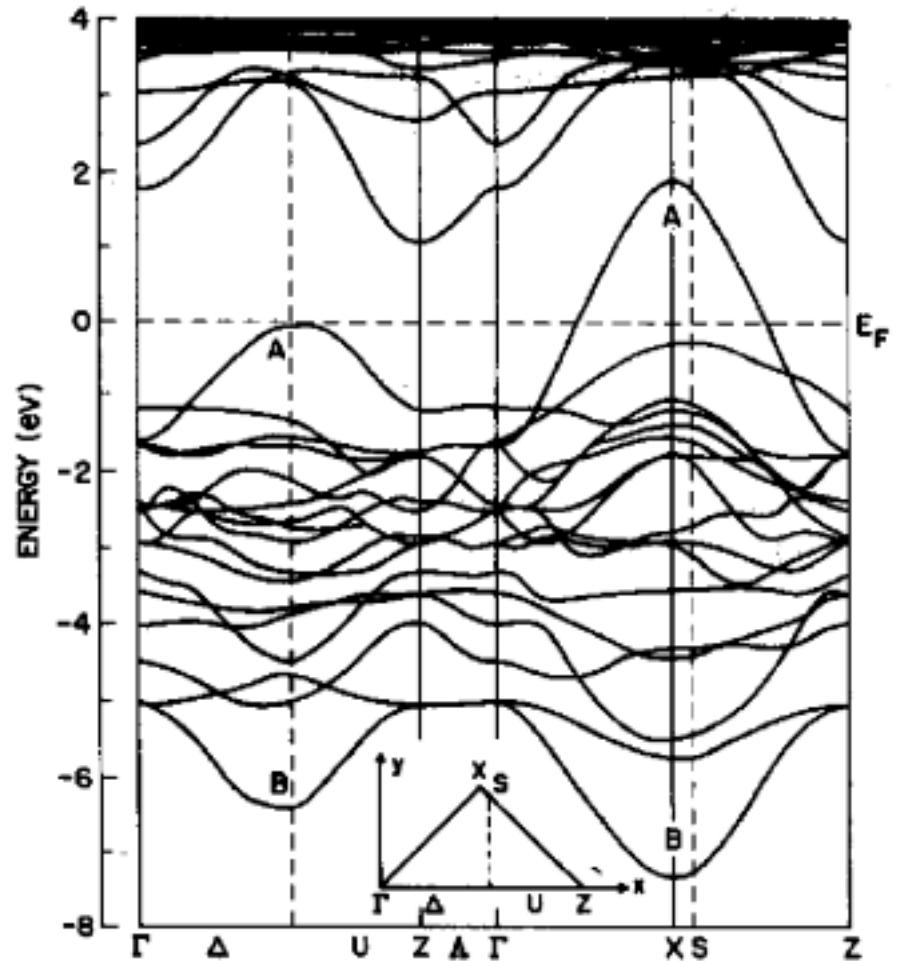


## The standard approaches :

- A. Quasiparticles, Fermi surface and Fermi liquids  
- LDA (Nobel prize 1998)

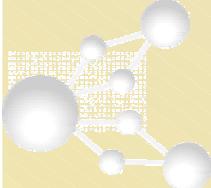
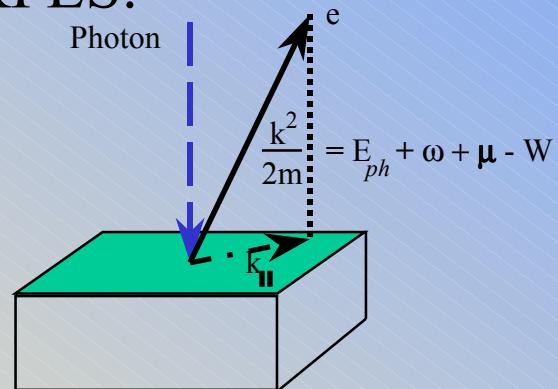


L.F. Mattheiss, Phys. Rev. Lett. 58, 1028 (1987).

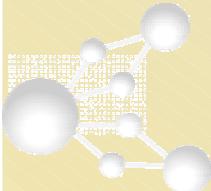
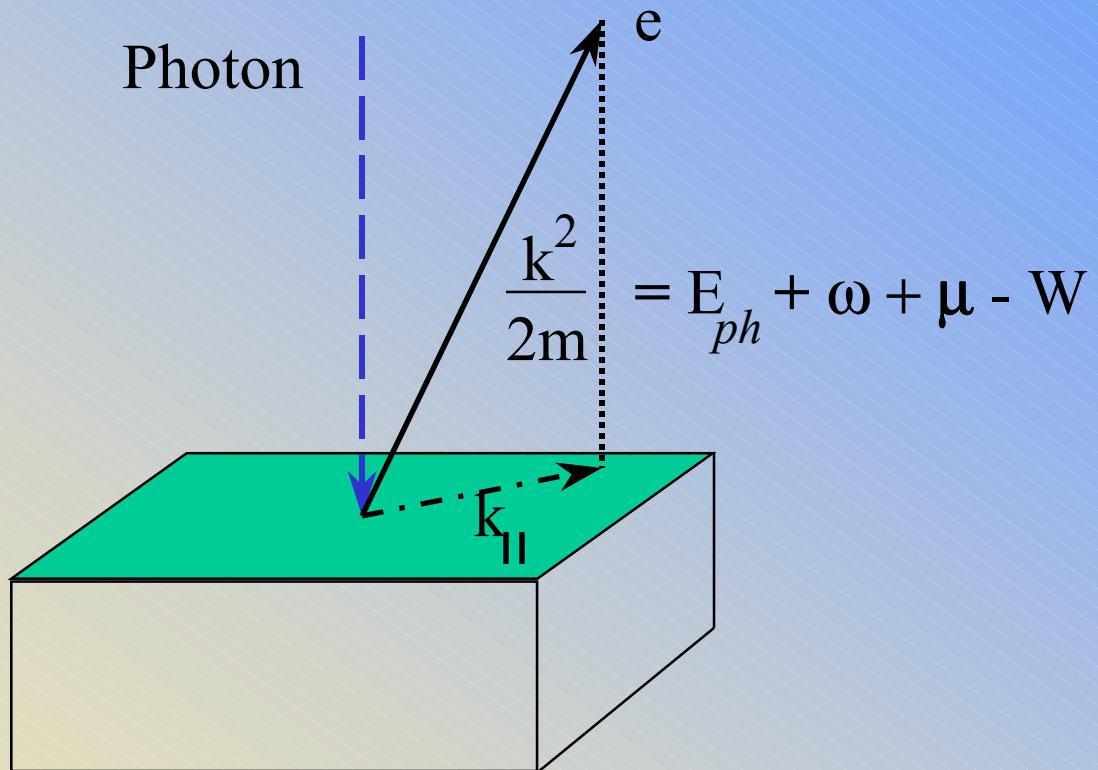


## The standard approaches :

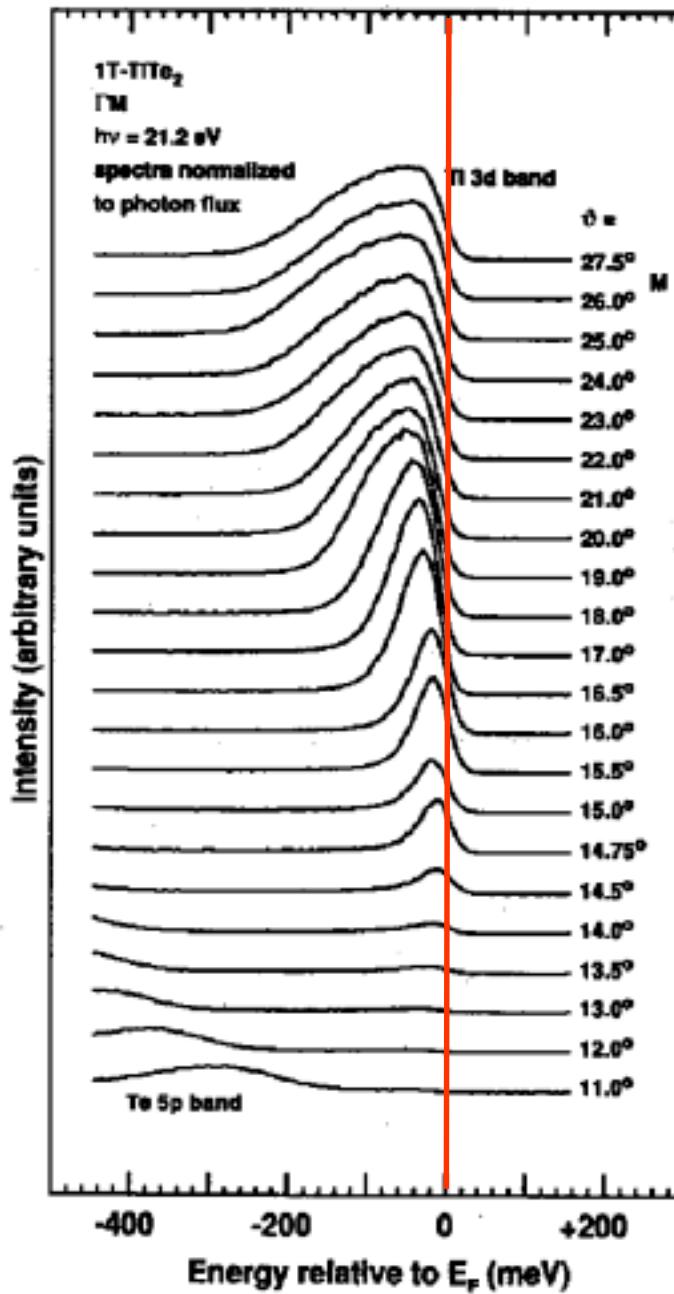
- Matrix elements of H in LDA basis
  - In practice, from general considerations:
    - Short range
    - Single Slater determinant not eigenstate
  - Phase space + Pauli restricts possible scatterings:
    - Quasiparticles  $m^*$ , effective fields,
- «See» the quasiparticles with ARPES:



## Photoemission



R. Claessen, R.O.  
Anderson, J.W. Allen,  
C.G. Olson, C.  
Janowitz, W.P. Ellis,  
S. Harm, M. Kalning,  
R. Manzke,  
and M. Skibowski,  
Phys. Rev. Lett. 69,  
808 (1992).



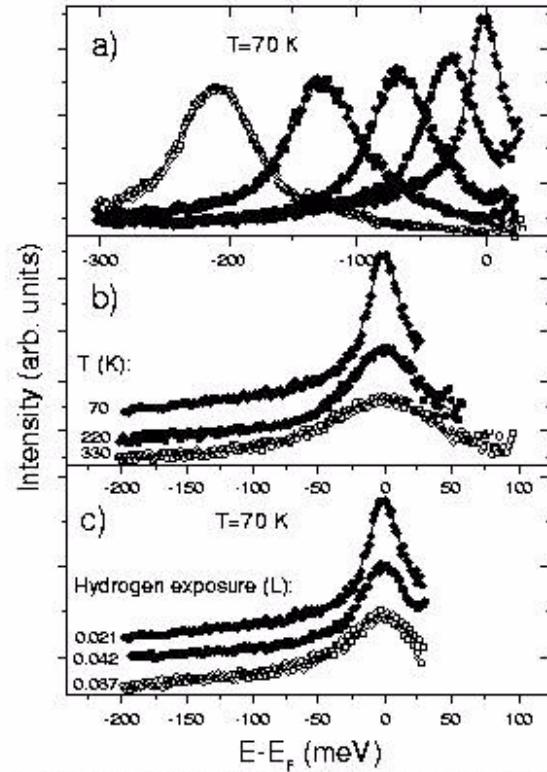


FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cut-off. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the binding energy (a), temperature (b), and hydrogen exposure (c) is shown.

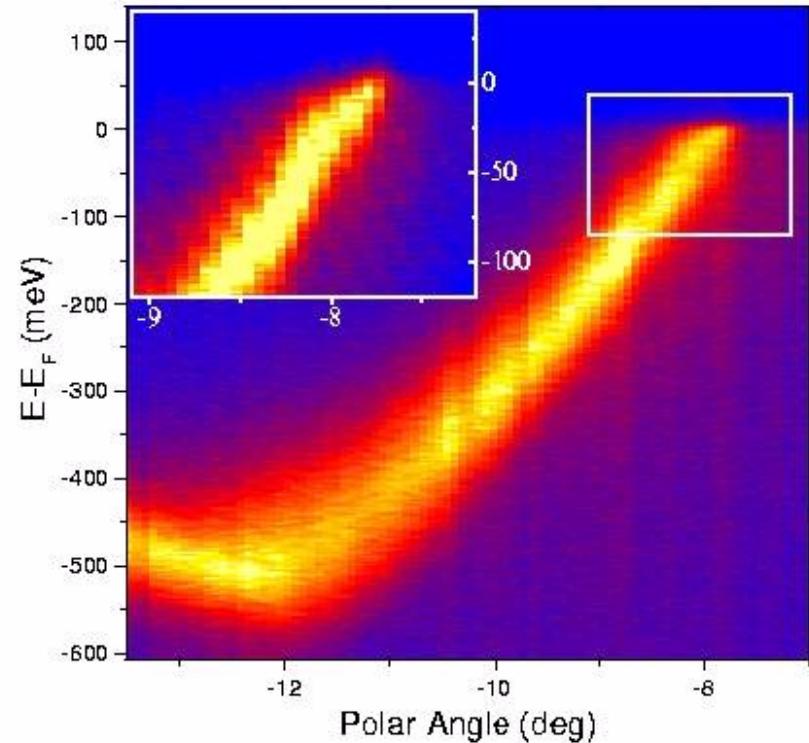


FIG. 1. ARPES intensity plot of the Mo(110) surface recorded along the  $\bar{\Gamma} - \bar{N}$  line of the SBZ at 70 K. Shown in the inset is the spectrum of the region around  $k_F$  taken with special attention to the surface cleanliness.

T. Valla, A. V. Fedorov, P. D. Johnson, and S. L. Hulbert  
P.R.L. 83, 2085 (1999).



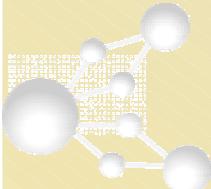
## The standard approaches :

### B. Thermodynamics and phase transitions

- Thermodynamics of Fermi liquids
  - particle-hole excitations
- Phase transitions
$$\chi \sim N(0) / (1 + F_o^a)$$
- Superconducting transition

### C. Heisenberg model and related models

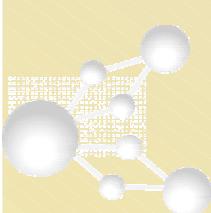
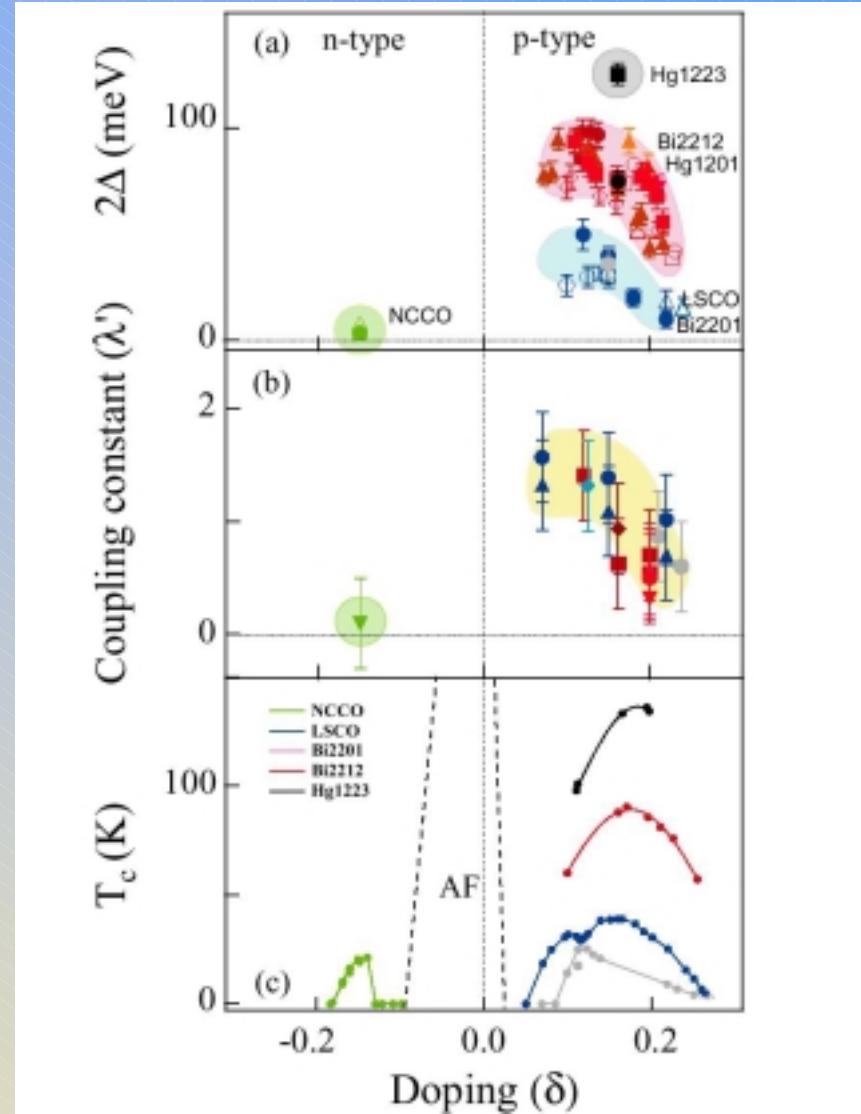
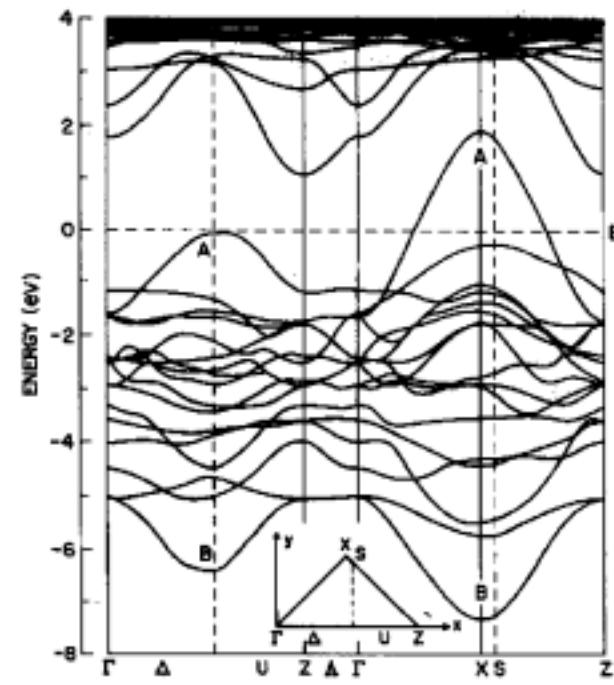
- Band for *s-p*
- Localized (often) for *d-f*
  - Only spin degrees of freedom
  - Use symmetry to write  $H$



## II. Failure of standard paradigm

$n = 1$ ,

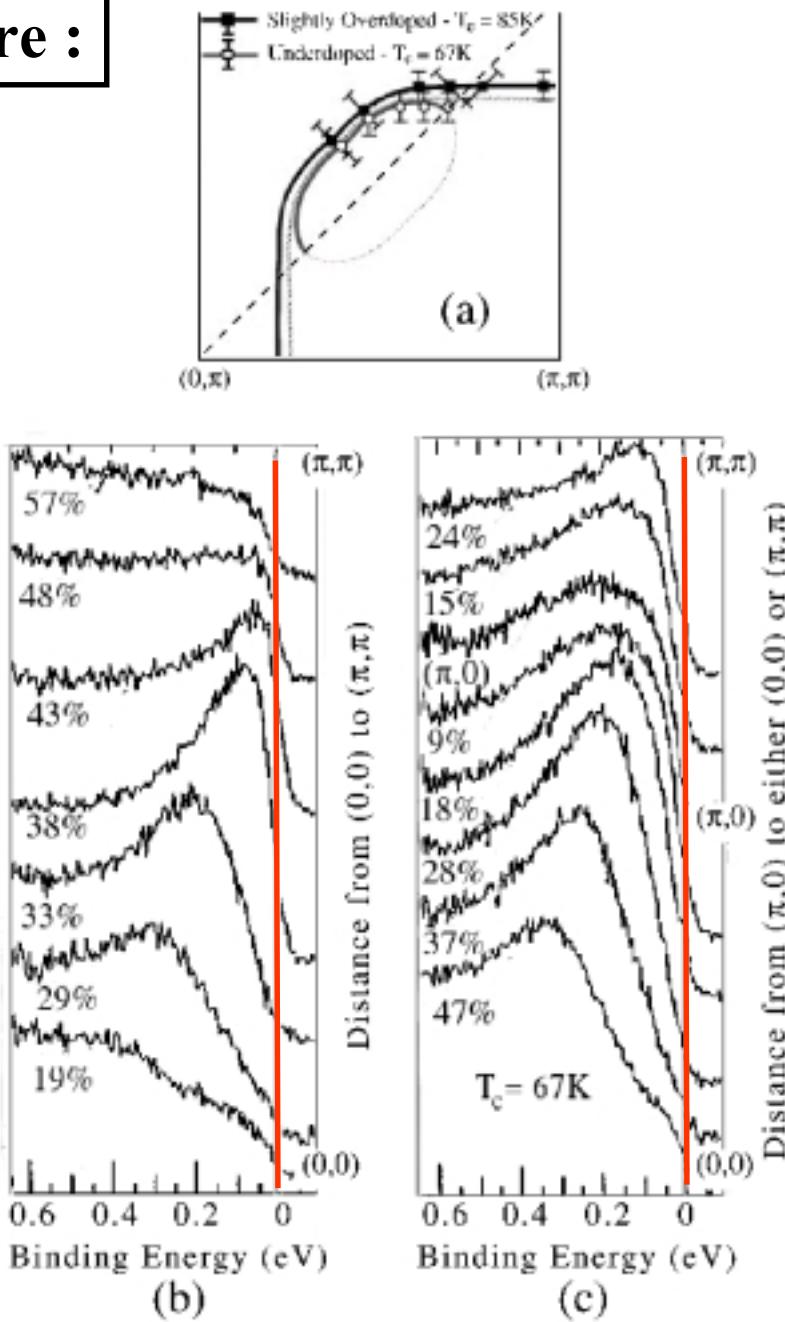
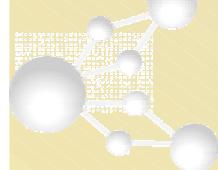
Metal according to band  
AFM insulator in reality



## Experimental evidence for failure :

-  $d=2$  partial vanishing act  
of the Fermi surface away  
from  $n = 1$ .

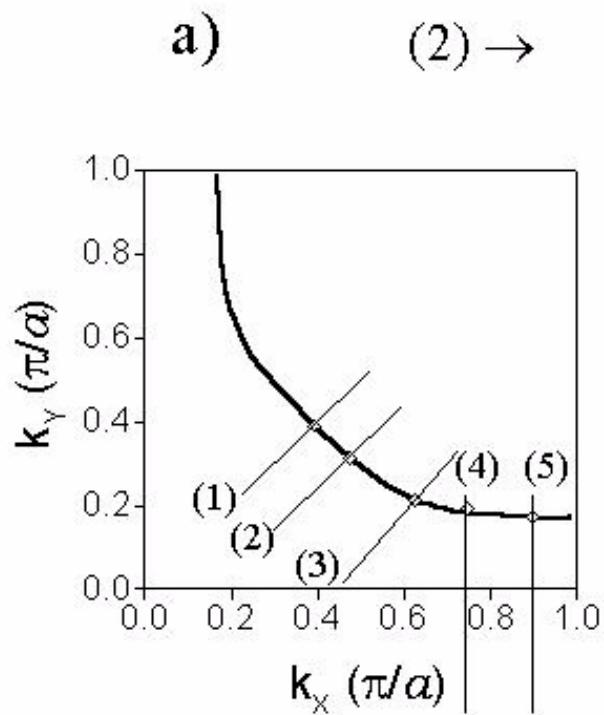
D.S. Marshall, D.S. Dessau, A.G.  
Loeser, C.-H. Park,  
A.Y. Matsuura, J.N. Eckstein, I.  
Bozovic, P. Fournier,  
A. Kapitulnik, W.E. Spicer, and  
Z.X. Shen, Phys. Rev.  
Lett. 76, 4841 (1996).



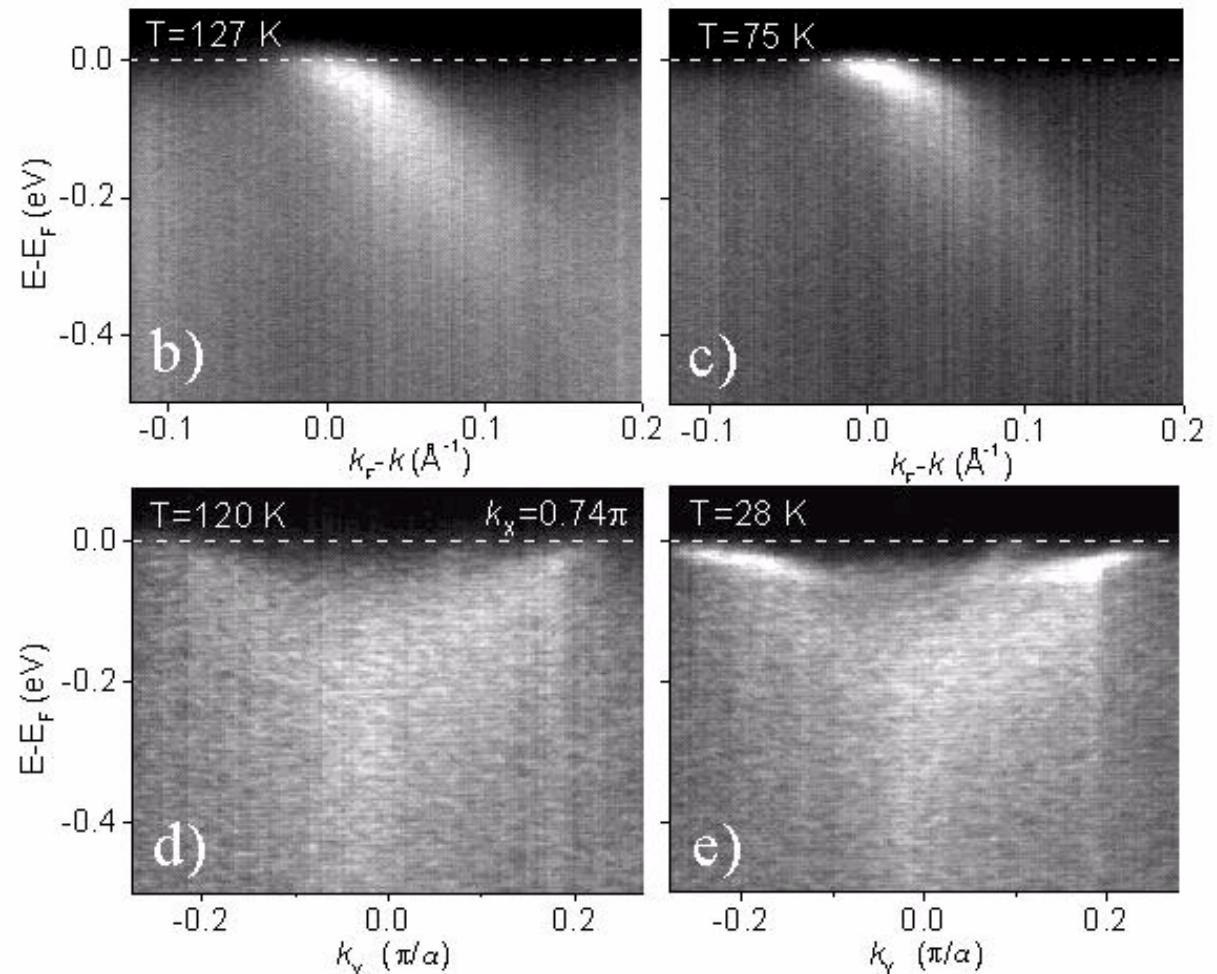
# Optimally doped BISCCO

Normal

Superconductor



(4) →



A. V. Fedorov, T. Valla, P. D. Johnson et al. P.R.L. **82**, 2179 (1999)

### III. Establishing a microscopic model

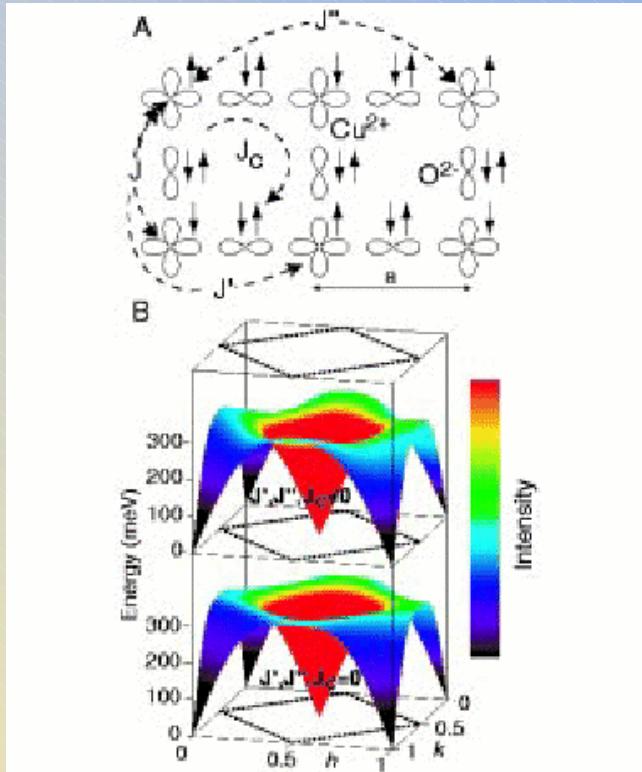


FIG. 1. (color) **A** The CuO<sub>2</sub> plane showing the atomic orbitals (Cu 3d<sub>x<sup>2</sup>-y<sup>2</sup></sub> and O 2p<sub>x,y</sub>) involved in the magnetic interactions.  $J$ ,  $J'$  and  $J''$  are the first-, second- and third-nearest-neighbor exchanges and  $J_c$  is the cyclic interaction which couples spins at the corners of a square plaquette. Arrows indicate the spins of the valence electrons involved in the exchange. **B** Lower surface is the dispersion relation for  $J=136$  meV and no higher-order magnetic couplings or quantum corrections. The upper surface shows the effect of the higher-order magnetic interactions determined by the present experiment. Color is spin-wave intensity.

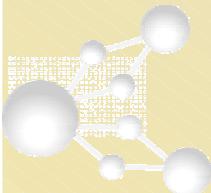
$$n = 1$$

Low energy excitations, spin waves are detected by neutron scattering.

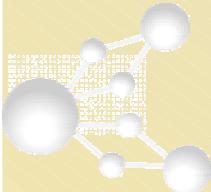
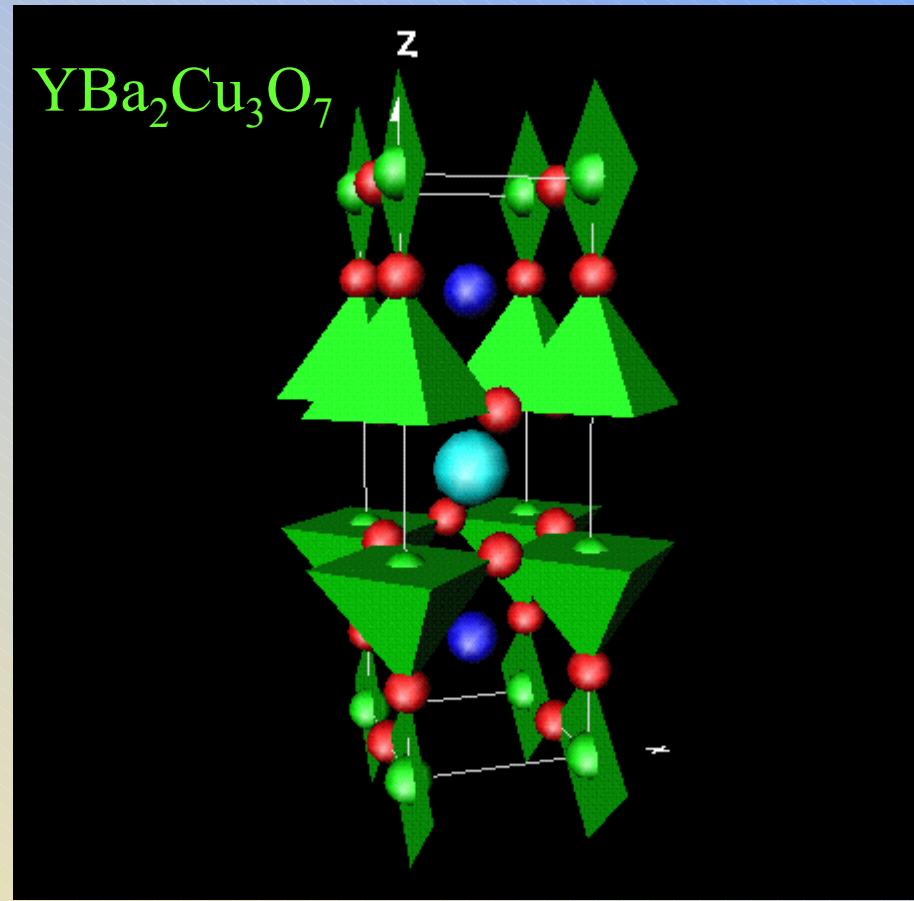
They are bidimensional.

← Experimental spin-wave dispersion

← Heisenberg,  $H = J \sum \mathbf{S}_i \cdot \mathbf{S}_j$



## Unit cell



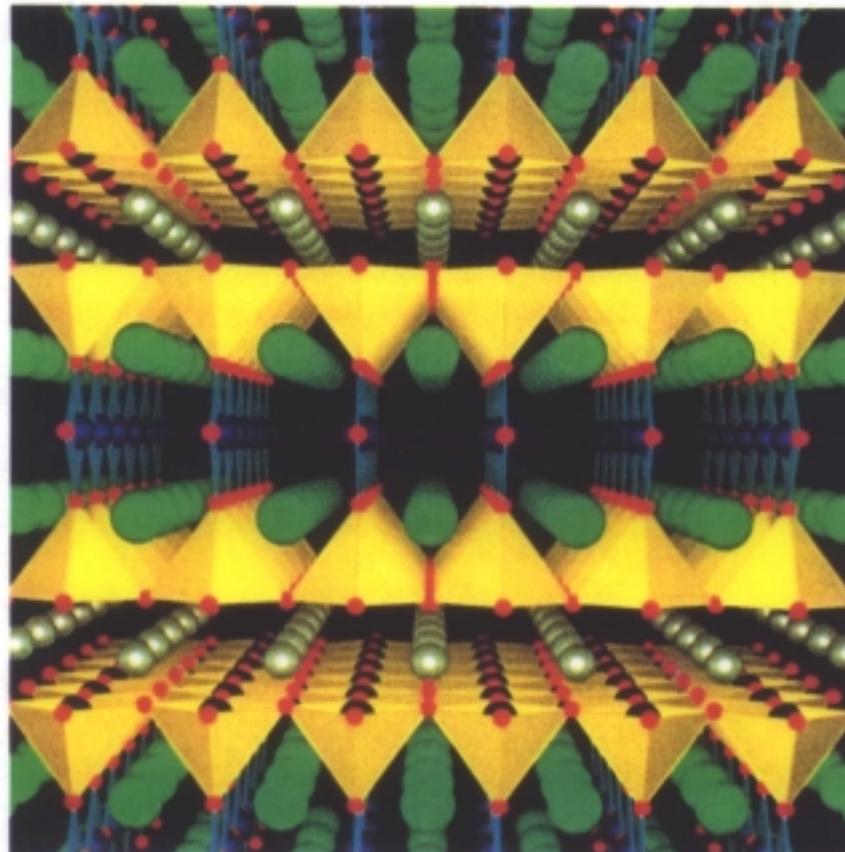
# SCIENTIFIC AMERICAN

JUNE 1988  
\$3.50

*How nonsense is deleted from genetic messages.*

*R for economic growth: aggressive use of new technology.*

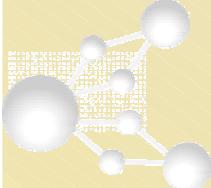
*Can particle physics test cosmology?*



*High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.*

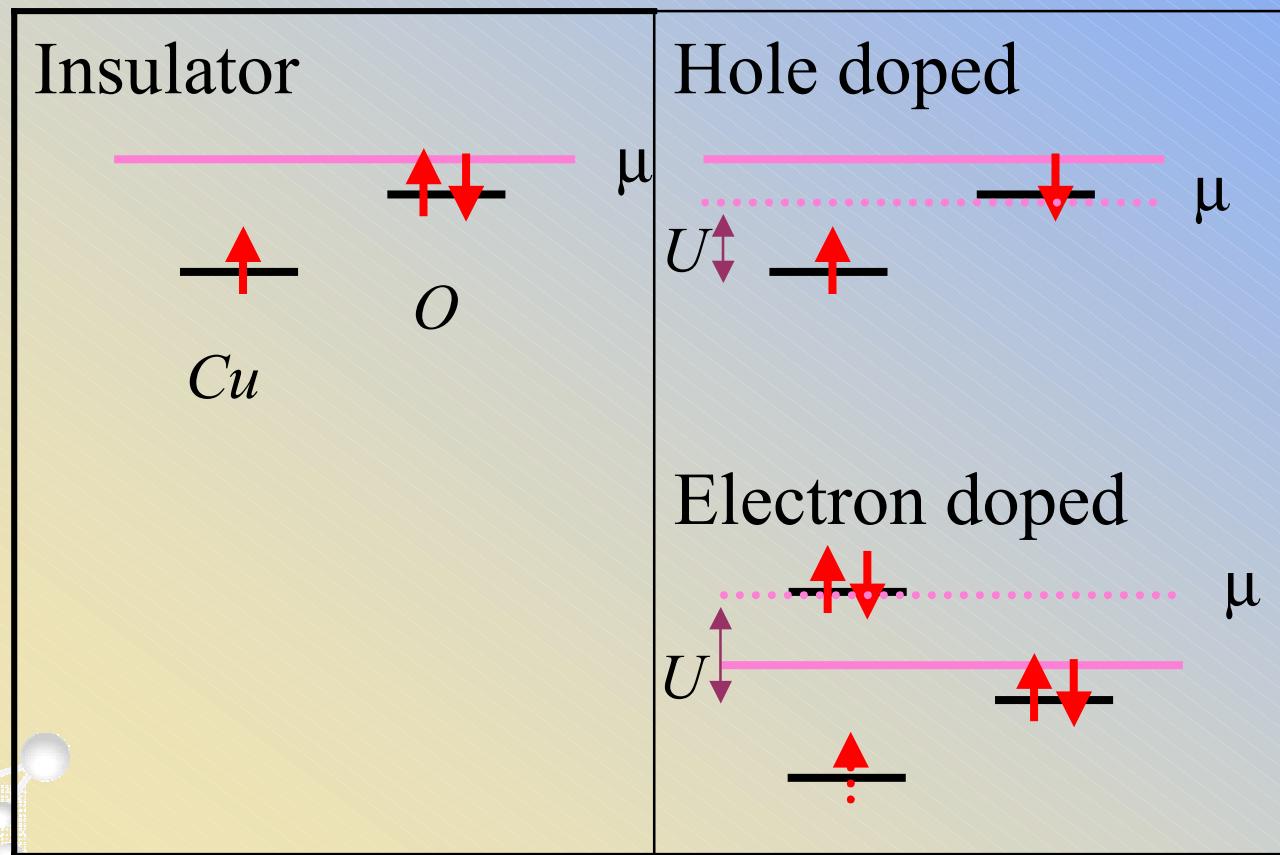
$\gamma\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$

92-37



- Atomic Physics  $CuO_2$  plane.
- Cu is  $3d^{10}4s^1$ .
- In high  $T_c$ ,  $Cu^{++}$ 
  - $3d_{x^2-y^2}$  has one hole
  - Four other levels are filled

*Zhang-Rice singlet*



*Charge transfer insulator*

## Insulator

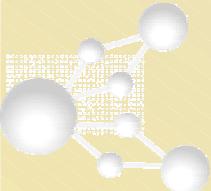


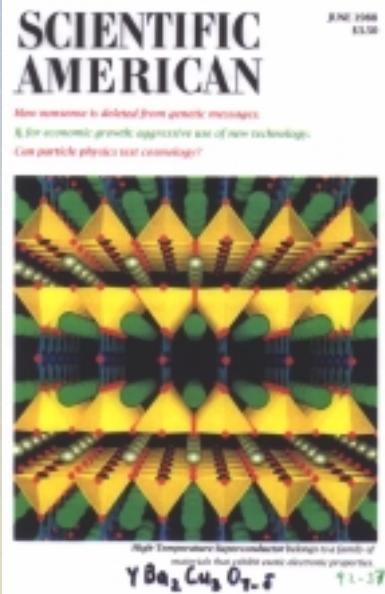
## Doped system : Doping $\delta$



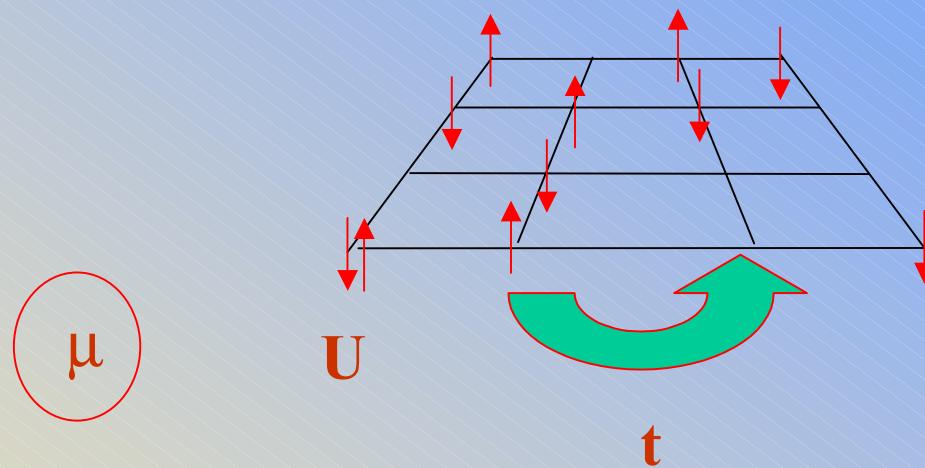
- UHB loses  $\delta$  states
- LHB gains  $2\delta$  states

Observable experimentally (XPS)

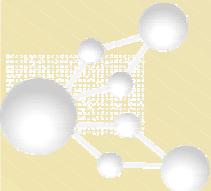




## Simplest microscopic model for $Cu\ O$ planes.



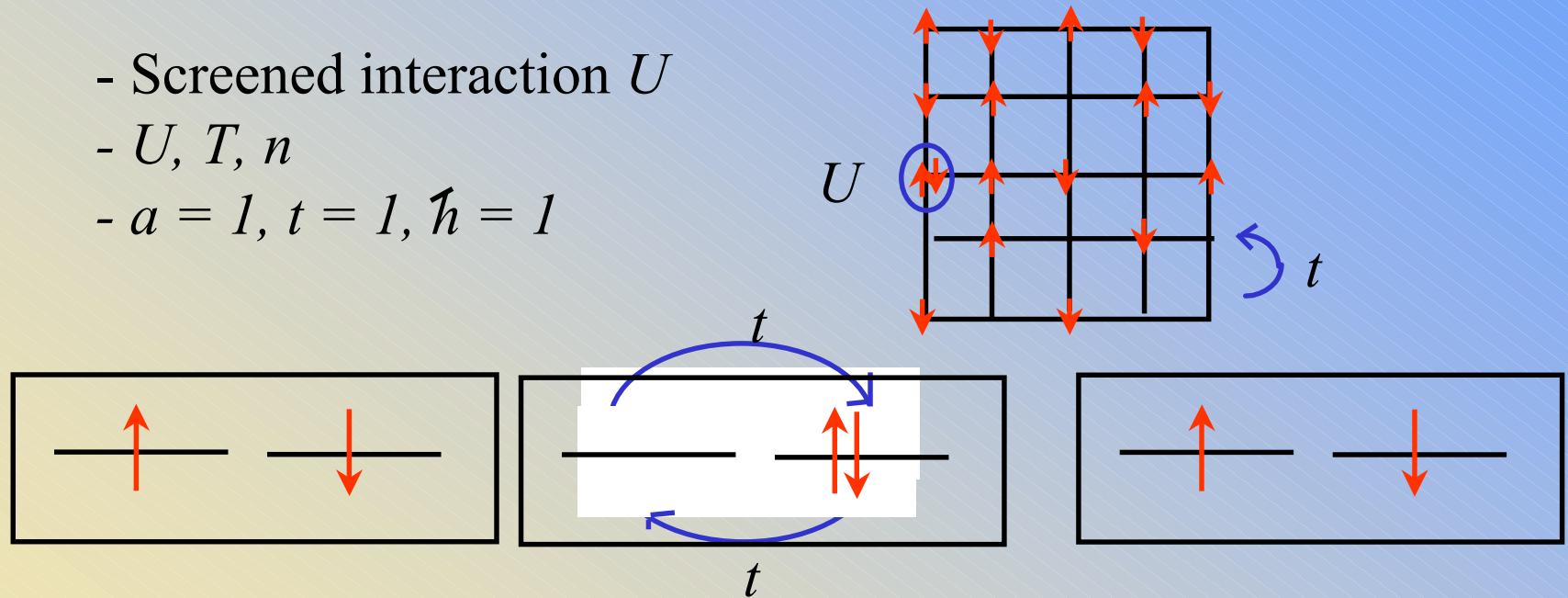
- Size of Hilbert space :  $4^N$  ( $N = 16$ )
- Compute 
$$\frac{Tr[Oe^{[-H/k_B T]}]}{Tr[e^{[-H/k_B T]}]}$$



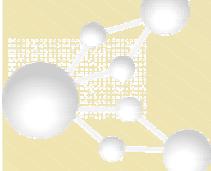
## Hubbard model (Kanamori, Gutzwiller, 1963) :

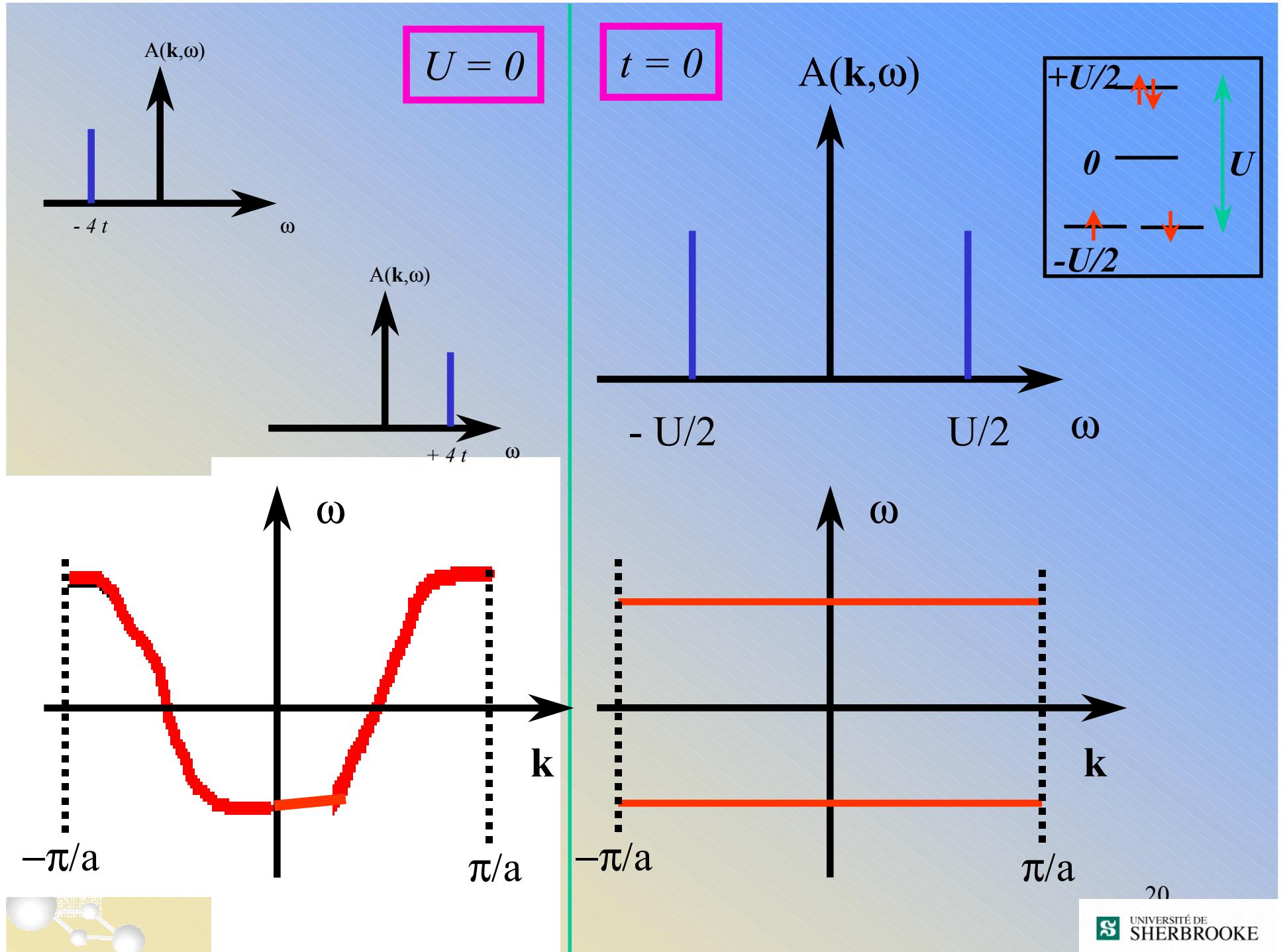
$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- Screened interaction  $U$
- $U, T, n$
- $a = 1, t = 1, \hbar = 1$

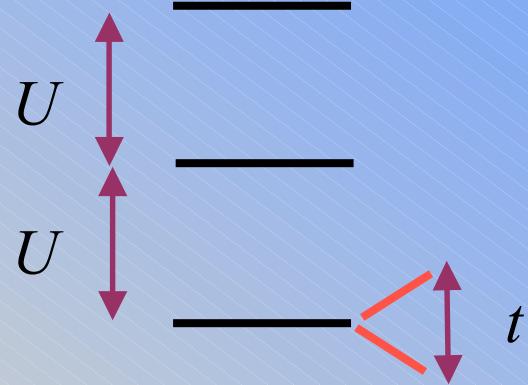


- 2001 vs 1963: Numerical solutions to check analytical approaches





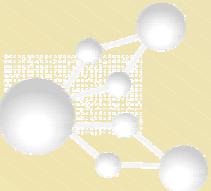
•  $t$ - $J$



- Number of states :  $3^N$

## • Problems

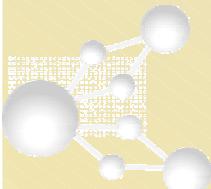
- Poor screening
- Phonons



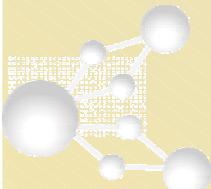
## IV. Theoretical difficulties

### (a) Straight numerical approaches

- $2^{30} \sim 1$  GB. Compare with  $4^{16}$  for 16 site lattice!
- Exact diagonalizations of  $t$ - $J$  for one hole suggest:
  - One peak in  $A(\mathbf{k}, \omega)$  plus incoherent background
  - Peak disperses with width of  $J$ . Thus
    - Number of carriers small.
    - Effective mass finite.
- Cluster Perturbation Theory (New method)
  - D. Senechal, D. Perez, M. Pioro-Ladrière Phys. Rev. Lett. 84 (2000) 522.



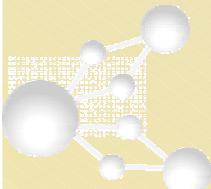
- Quantum Monte Carlo (S.R. White *et al.* Phys. Rev. B **40**, 506 (1989))
  - AFM at  $n = 1$ . Away from half-filling problems with BC
  - Pseudogap in weak to intermediate coupling
  - Temperature not low enough to establish *d-wave* superconductivity
  - Useful as a benchmark for analytical approaches.
- Main drawback: « fermion sign problem » and instabilities
- Density Matrix Renormalization Group
  - People are working on  $d = 2$  generalizations.



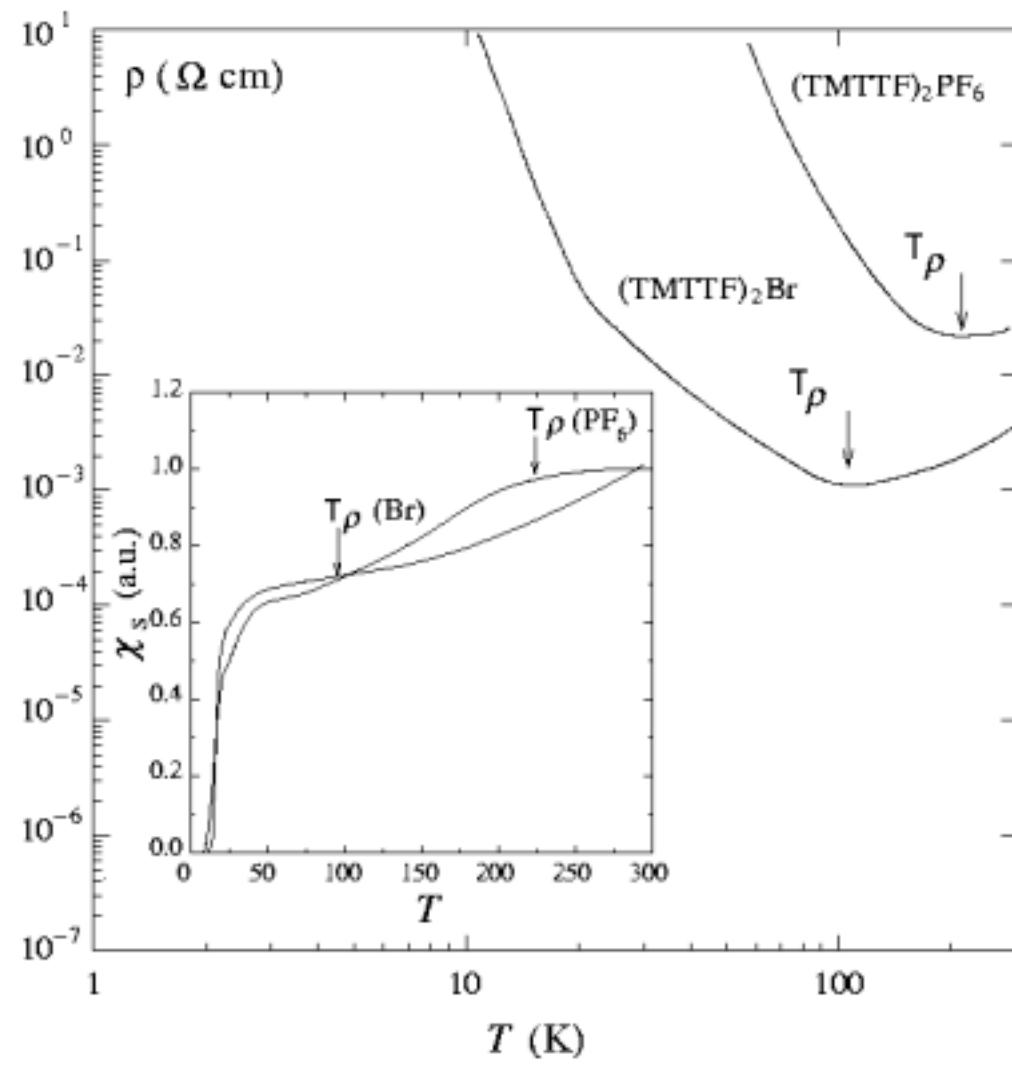
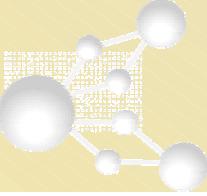
## (b) Inadequacy of mean field in low dimension

- Fluctuations dominate the physics
  - $d = 1$  Spin-Charge separation (*Luttinger* liquid Behavior)  
(Quantum fluctuations at  $T = 0$ )
  - $d = 2$  Thermal fluctuations :
    - *Hohenberg-Coleman-Mermin-Wagner* theorem.

And the strong interactions complicate all that.



C. Bourbonnais  
et al.  
(cond-  
mat/9903101).



-  $d=1$  spin-charge separation

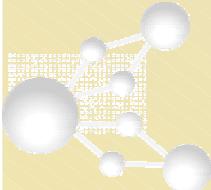
- Straight perturbation theory for nested  $d=2$  Fermi surface gives non-trivial results (F. Lemay PhD thesis, 2000)
- Thermal and quantum fluctuations in  $d = 2$

$d = 2$ , Mermin-Wagner

$$(\nabla\theta)^2 \rightarrow q^2\theta_{\mathbf{q}}\theta_{-\mathbf{q}}$$

$$\langle\theta^2\rangle \propto \int d^2q \frac{kT}{q^2} \rightarrow \infty$$

$d = 1$  : R.G., Bosonization, Conformal Field Theory...  
 $d = 2$  : Slave bosons, R.G., strong coupling p.t., TPSC

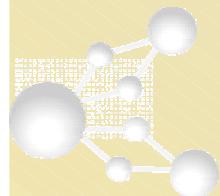


## Pseudogap

- New Ising character phase?
  - RVB
  - Preformed pairs
  - Flux phase
  - D Density Wave
- Fluctuations?
  - SC, AFM, singlet...

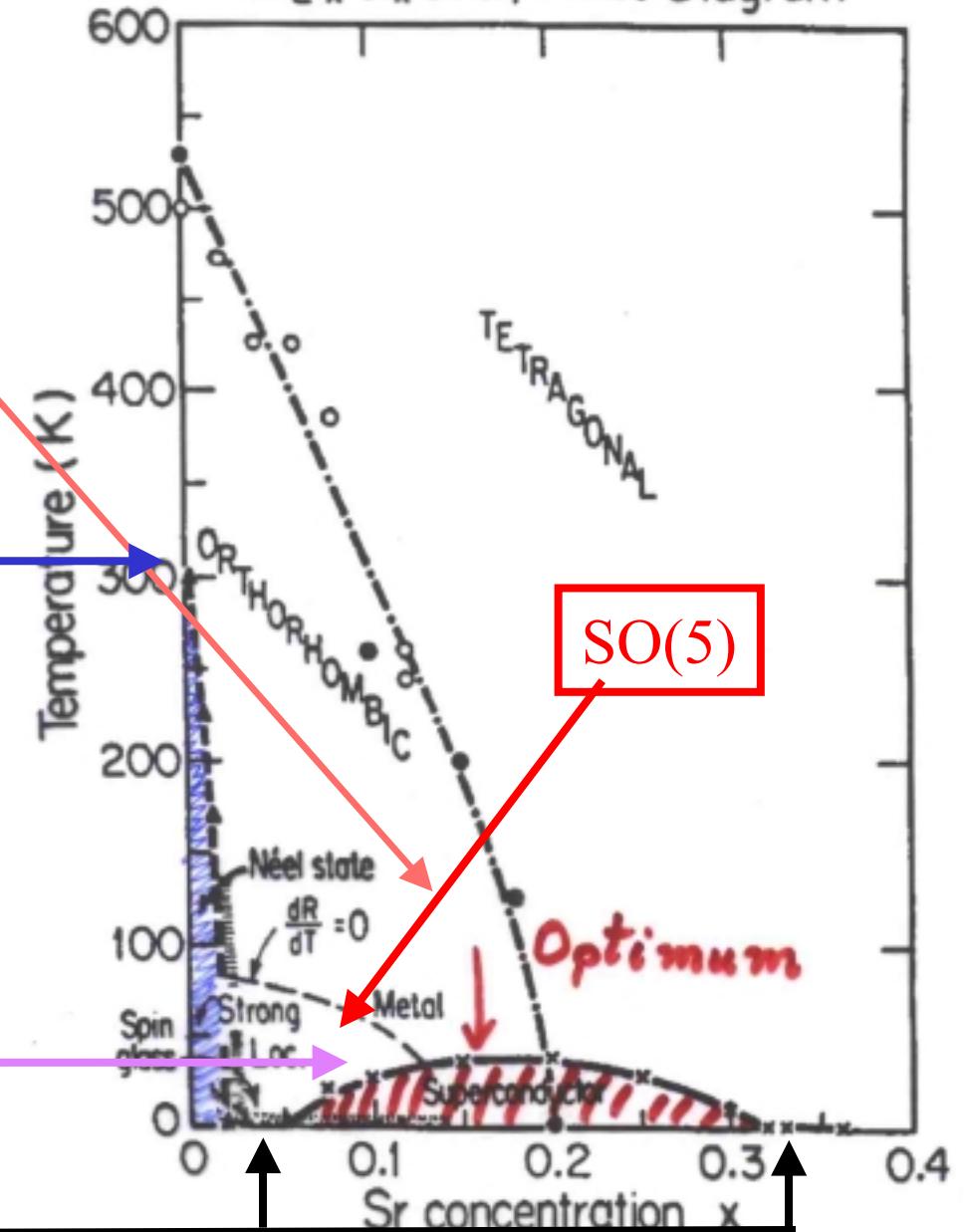
$d = 3$  Néel  $T$

« Mott » Physics explains  
decreasing  $T_c$   
(Small  $\rho_s$ , Phase fluctuations)



Quantum critical points

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  Phase Diagram



Would help decide  
« What type » of fluctuations  
are important. (See later)

Loram and Tallon, cond-mat/0005063

See Sonier at CIAR conf.

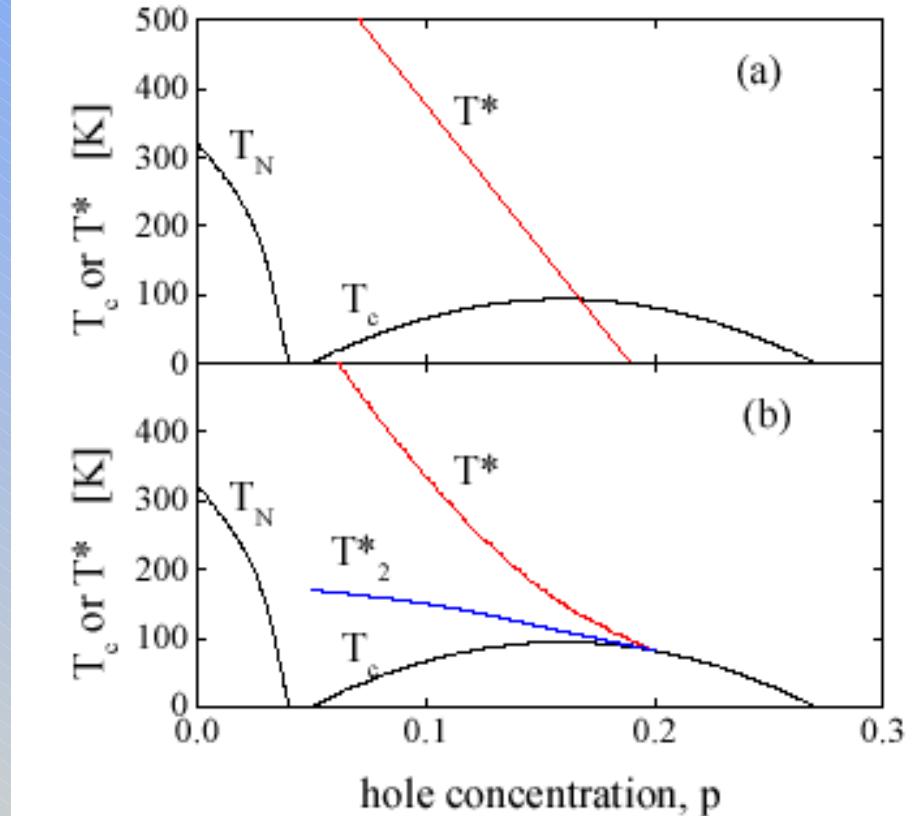
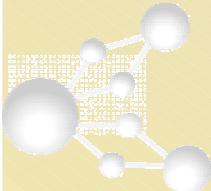
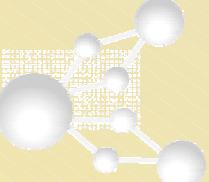


Fig. 1. Two scenarios for the “phase diagram” for HTS cuprates. In (a)  $T^*$  represents an energy scale which falls abruptly to zero at a critical doping,  $p=0.19$ . In (b)  $T^*$  merges with  $T_c$  on the overdoped side and often a lower  $T^{*2}$  associated with a small pseudogap or a spin gap is invoked.  $T_N$  is the Neel temperature for the 3-D AF state.



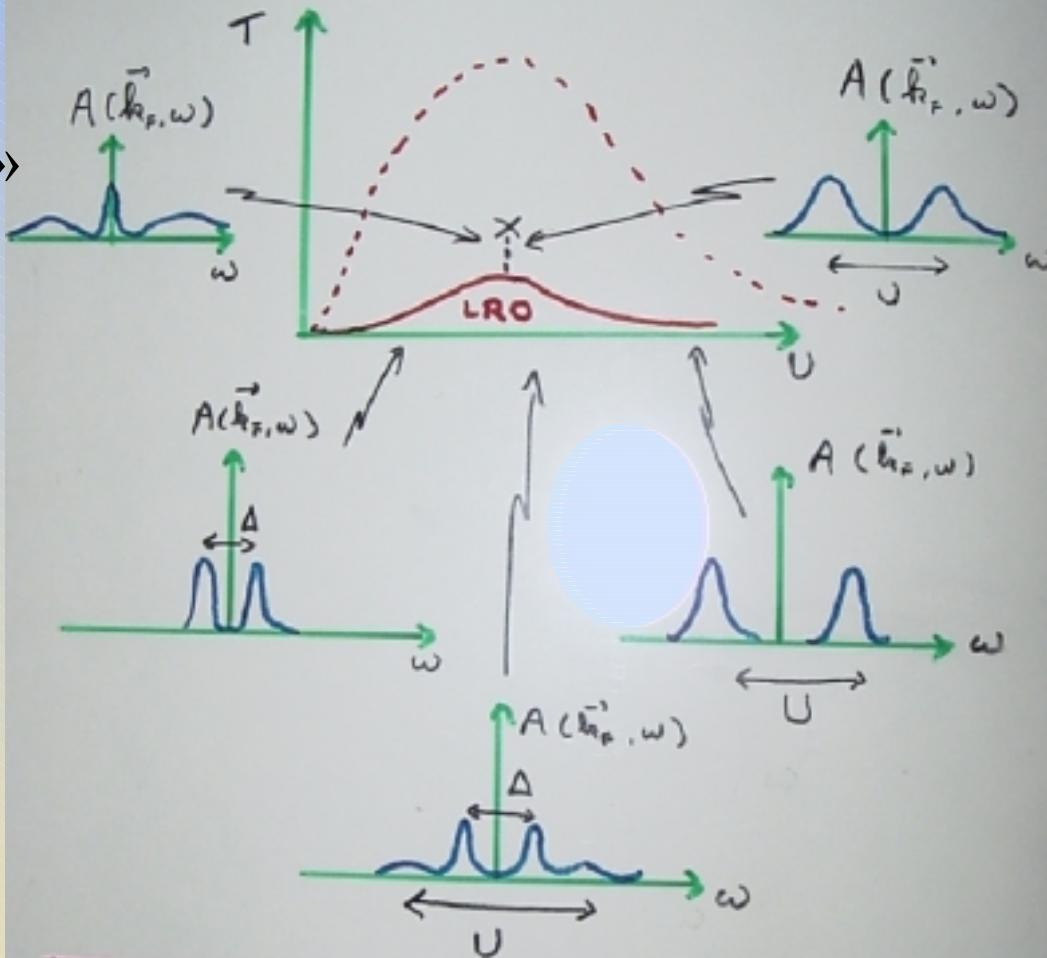


High  $T_c$  are at « intermediate » coupling:

$$U = 8t \\ t \sim 300 \text{ meV}$$

R. Coldea  
cond-mat/ 0006384

## "Weak" vs "Strong" coupling ( $\omega = \infty$ )



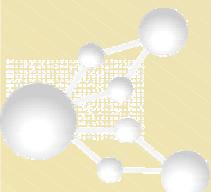
### Weak to intermediate coupling:

- Perturbation theory?  
(RPA violates Pauli, to  $O(U/\omega)^2$ )
- Self-consistent approaches
- Are Fermi-liquid parameters calculable?

## (c) Approaching from weak coupling

- Perturbation theory (Wick's theorem)
  - Finding « effective interactions »
    - RPA with long range interaction *vs* Hubbard
  - Notion of « channel »
  - RG and « interference »
- 
- Problem with Pauli principle (Parquet, *Bickers*)
  - Self-consistent treatments
  - Migdal's theorem *vs* theory of « ordinary » superconductors
- 
- Alternative: satisfy sum-rules
    - Conservation laws
    - Pauli principle
    - Mermin-Wagner theorem

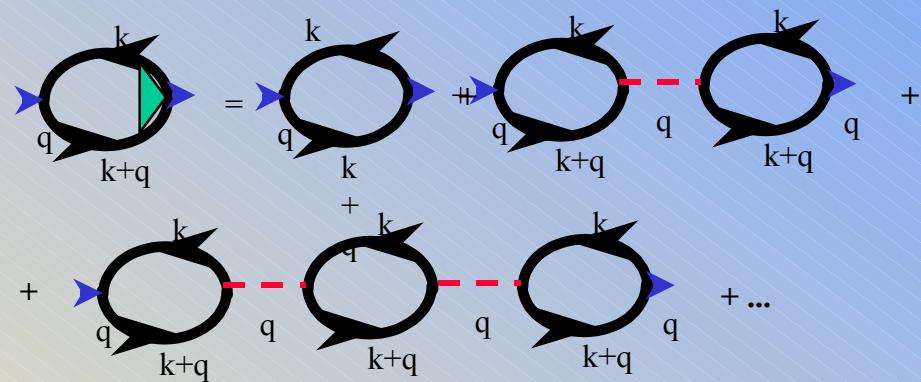
*Fluctuation-induced pseudogap?*



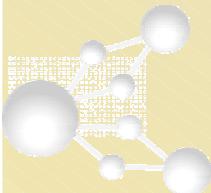
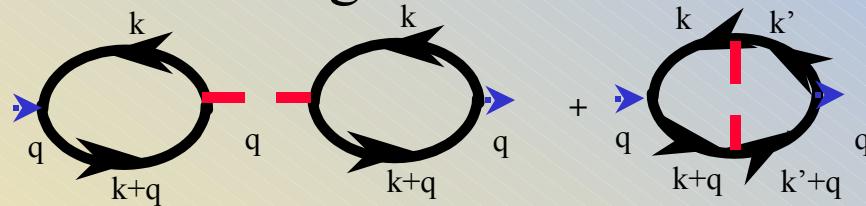
- Perturbation theory:

$$\frac{1}{(X + Y)} = \frac{1}{X} - \frac{1}{X} \frac{1}{Y} \frac{1}{X + Y}.$$

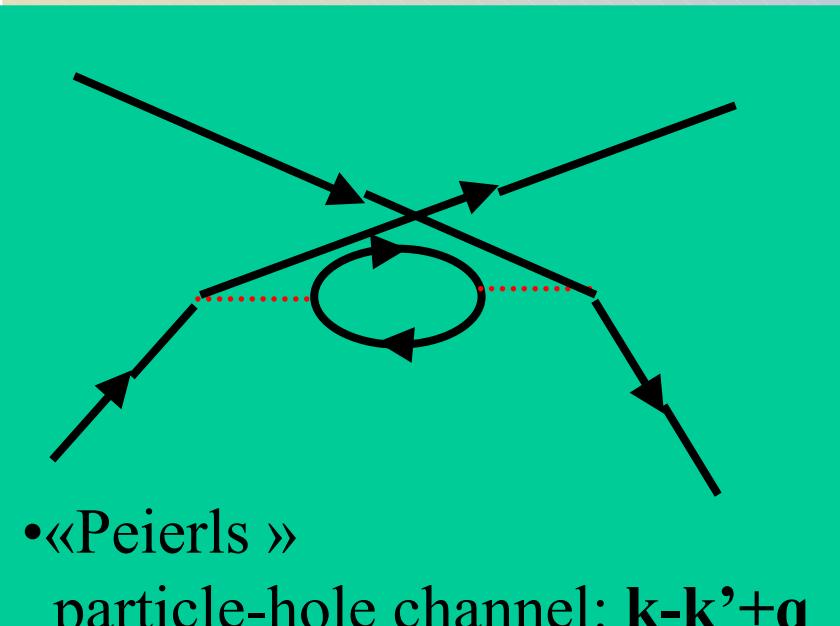
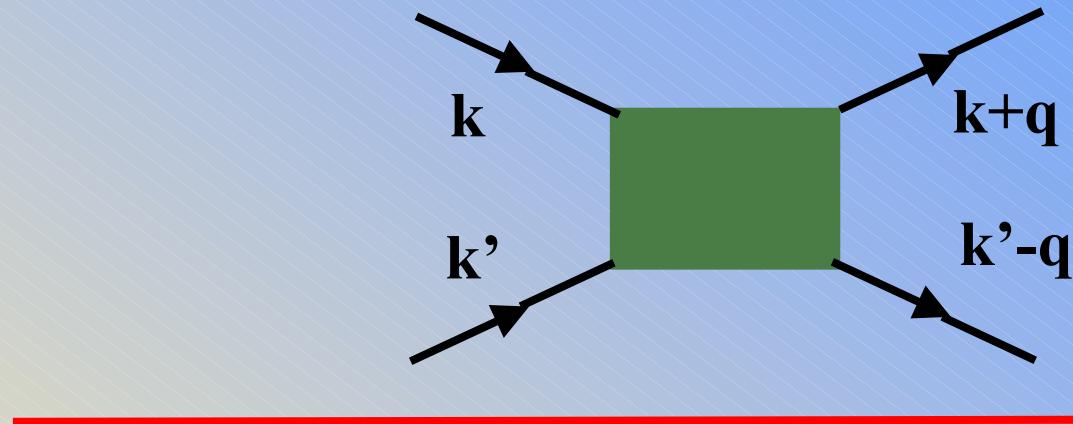
- Effective interaction (Screening in usual solids)



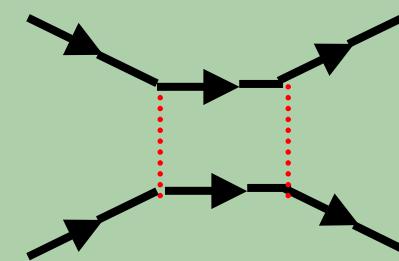
- Problem with « short-range »



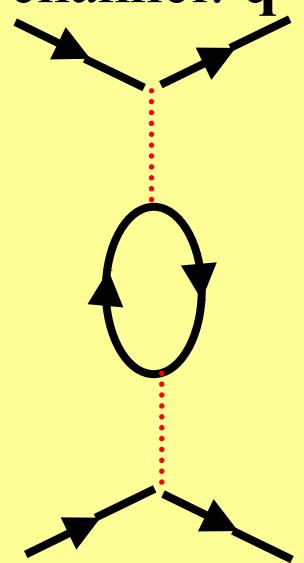
- Notion of « channel »



- Particle – particle  
channel:  $k + k'$



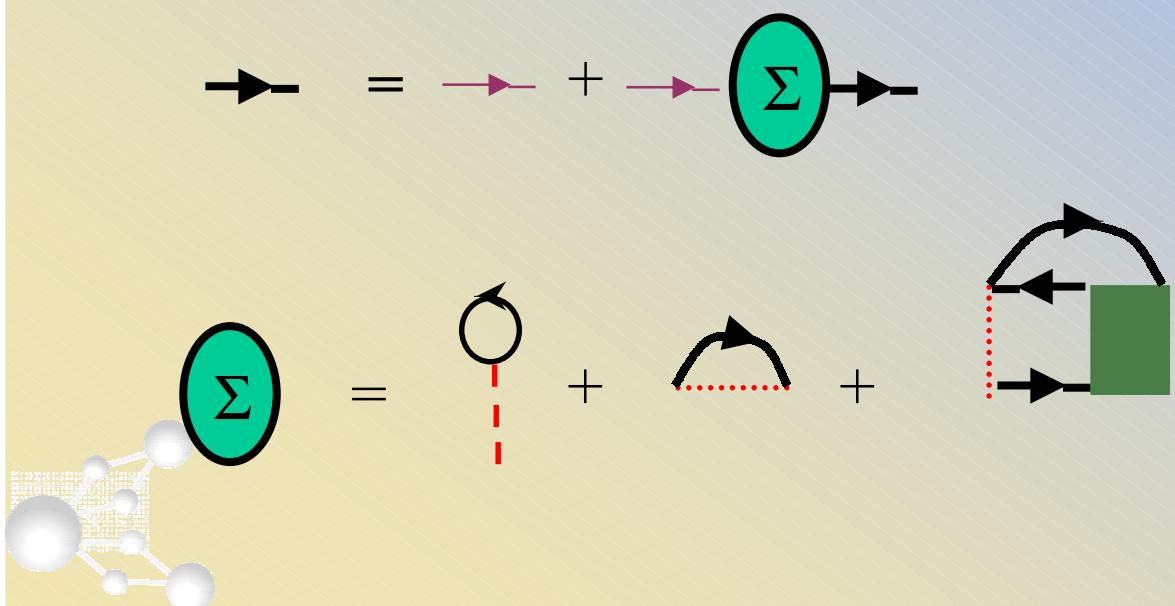
- « Landau »  
p-h channel:  $q$



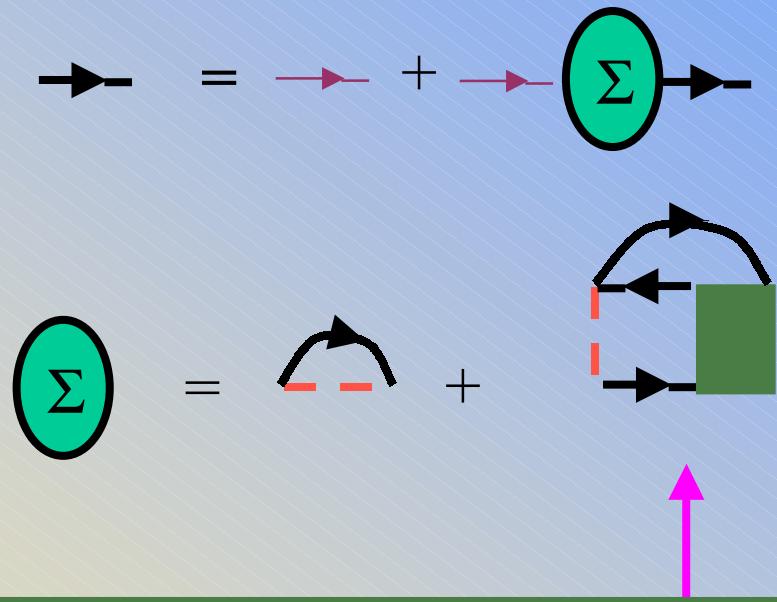
- Renormalization group : « interference »

Zanchi, Schultz... Recent: C. Honerkamp *et al.* Phys. Rev. B **63** 035109 (2001)

- Problem with Pauli principle when summing infinite sets of diagrams
  - Need to include all crossed diagrams in practice. Impossible
  - Can be done approximately with Parquet approximation (Bickers) (with self-consistency, unsatisfactory)
- « Self-consistency » and conservation laws



- Self-consistency in conventional « Eliashberg » superconductivity
  - Take phonons as « given »



$$\Sigma = \delta F / \delta G$$

$$\Gamma_{\text{irr}} = \delta \Sigma / \delta G$$

Conservation laws, but:  
no Pauli, infinite number of theories, assumes Migdal

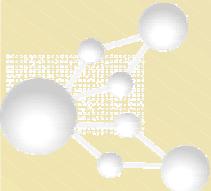
**Migdal's theorem:**

can be dropped because  $(m/M)^{1/2} \ll 1$

Not so for spin-fluctuation exchange

Non perturbative but from weak coupling:

- Pauli
  - Conservation laws
  - Mermin-Wagner
- 
- Main result: Fluctuation-induced pseudogap



# How it works...

## First step: Two-Particle Self-Consistent

---

$$\Sigma_{\sigma}^{(1)}(1, \bar{1}) G_{\sigma}^{(1)}(\bar{1}, 2) = A G_{-\sigma}^{(1)}(1, 1^+) G_{\sigma}^{(1)}(1, 2)$$

where  $A$  depends on external field and is chosen such that the exact result

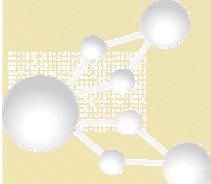
$$\Sigma_{\sigma}(1, \bar{1}) G_{\sigma}(\bar{1}, 1^+) = U \langle n_{\uparrow} n_{\downarrow} \rangle$$

is satisfied. One finds

$$A = U \frac{\langle n_{\uparrow} n_{\downarrow} \rangle}{\langle n_{\uparrow} \rangle \langle n_{\downarrow} \rangle}$$

Functional derivative of  $\langle n_{\uparrow} n_{\downarrow} \rangle / (\langle n_{\uparrow} \rangle \langle n_{\downarrow} \rangle)$  drops out of spin vertex

$$U_{sp} = A = U \frac{\langle n_{\uparrow} n_{\downarrow} \rangle}{\langle n_{\uparrow} \rangle \langle n_{\downarrow} \rangle}$$



## How it works...

To close the system of equations, while satisfying conservation laws and the Pauli principle

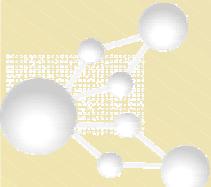
$$\begin{aligned} \left\langle (n_{\uparrow} - n_{\downarrow})^2 \right\rangle &= \langle n_{\uparrow} \rangle + \langle n_{\downarrow} \rangle - 2 \langle n_{\uparrow} n_{\downarrow} \rangle \\ \boxed{\frac{T}{N} \sum_{\tilde{q}} \frac{\chi_0(q)}{1 - \frac{1}{2} U_{sp} \chi_0(q)}} &= n - 2 \langle n_{\uparrow} n_{\downarrow} \rangle \end{aligned} \quad (1)$$

Recall

$$U_{sp} = U \frac{\langle n_{\uparrow} n_{\downarrow} \rangle}{\langle n_{\uparrow} \rangle \langle n_{\downarrow} \rangle} \quad (2)$$

To have charge fluctuations that satisfy Pauli principle as well,

$$\boxed{\frac{T}{N} \sum_q \frac{\chi_0(q)}{1 + \frac{1}{2} U_{ch} \chi_0(q)}} = n + 2 \langle n_{\uparrow} n_{\downarrow} \rangle - n^2 \quad (3)$$



(Bonus: Mermin-Wagner theorem)

How it works...

## Second step: improved self-energy

$$\Sigma_\sigma(1, \bar{1}) G_\sigma(\bar{1}, 2) = -U \left\langle \psi_{-\sigma}^\dagger(1^+) \psi_{-\sigma}(1) \psi_\sigma(1) \psi_\sigma^\dagger(2) \right\rangle_\phi$$

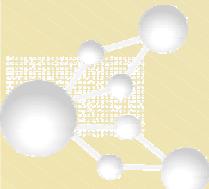
$$\Sigma_\sigma(1, \bar{1}) G_\sigma(\bar{1}, 2) = -U \left[ \frac{\delta G_\sigma(1, 2)}{\delta \phi_{-\sigma}(1^+, 1)} - G_{-\sigma}(1, 1^+) G_\sigma(1, 2) \right]$$

Last term is Hartree Fock ( $\lim \omega \rightarrow \infty$ ). Multiply by  $G^{-1}$ , replace lower energy part results of TPSC

$$\Sigma_\sigma^{(2)}(1, 2) = U G_{-\sigma}^{(1)}(1, 1^+) \delta(1 - 2) - U G^{(1)} \left[ \frac{\delta \Sigma^{(1)}}{\delta G^{(1)}} \frac{\delta G^{(1)}}{\delta \phi} \right]$$

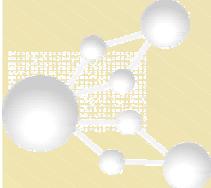
Transverse+longitudinal for crossing-symmetry

$$\boxed{\Sigma_\sigma^{(2)}(k) = U n_{-\sigma} + \frac{U}{8} \frac{T}{N} \sum_q [3U_{sp} \chi^{(1)}(q) + U_{ch} \chi^{(1)}(q)] G_\sigma^{(1)}(k + q). \quad (4)}$$



$$1 \xrightarrow[3]{\quad} 2 = -1 \xrightarrow[3]{\quad} 2 + 1 \xrightarrow[3]{\quad} \begin{matrix} \bar{2} \\ \bar{4} \end{matrix} \xrightarrow{\quad} 2$$

$$1 \cdot \Sigma \cdot 2 = 1 \xrightarrow[1]{\quad} 2 + 1 \xrightarrow[\frac{1}{2}]{\quad} \begin{matrix} \bar{5} \\ \bar{4} \end{matrix} \xrightarrow{\quad} 2$$



# A non-perturbative approach for both $U > 0$ and $U < 0$

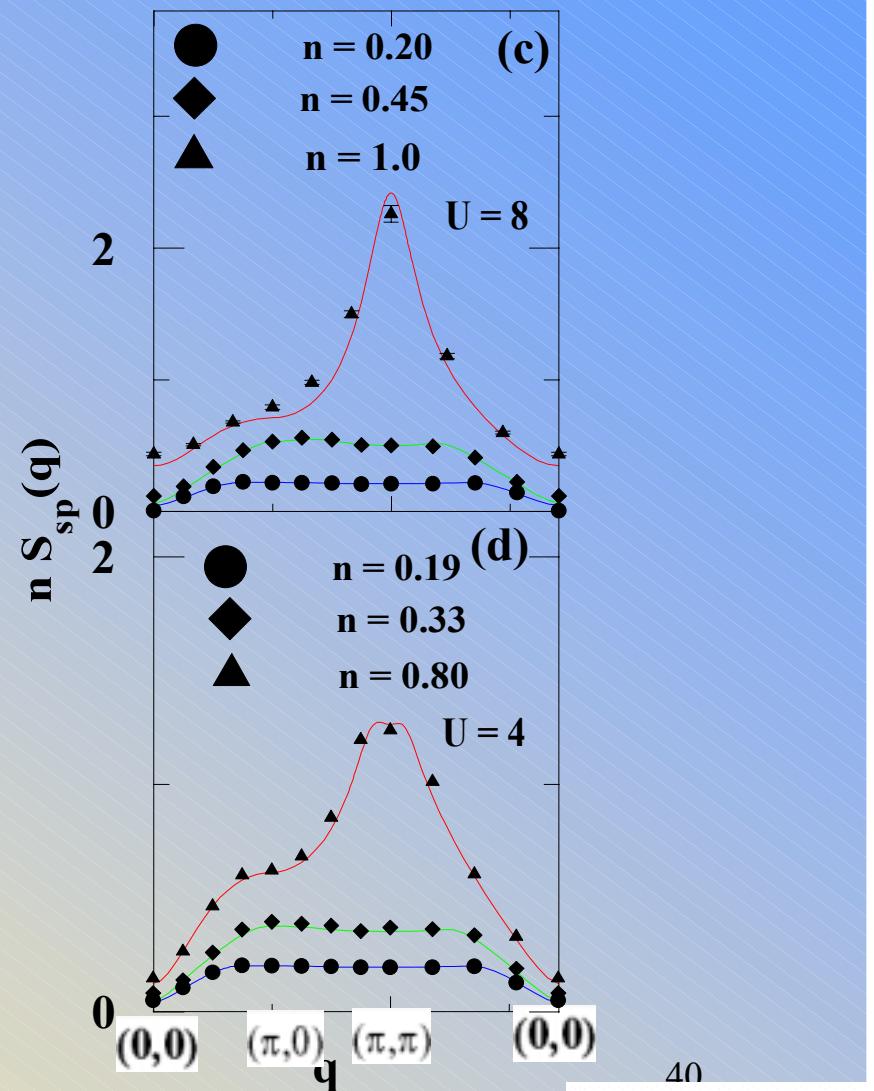
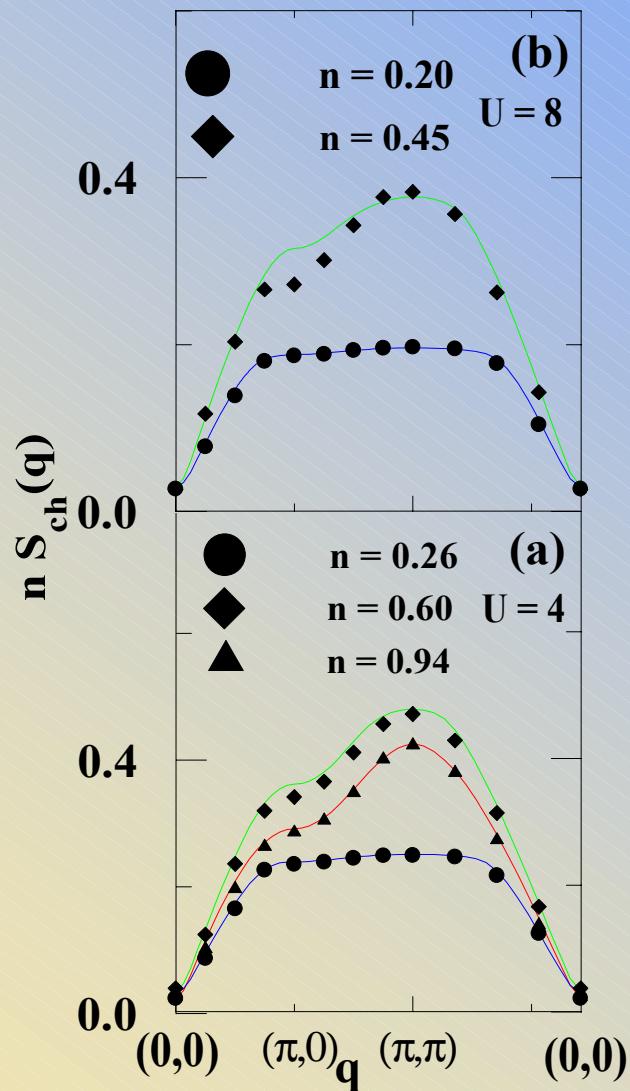
- Proofs that it works

$U > 0$

Notes:

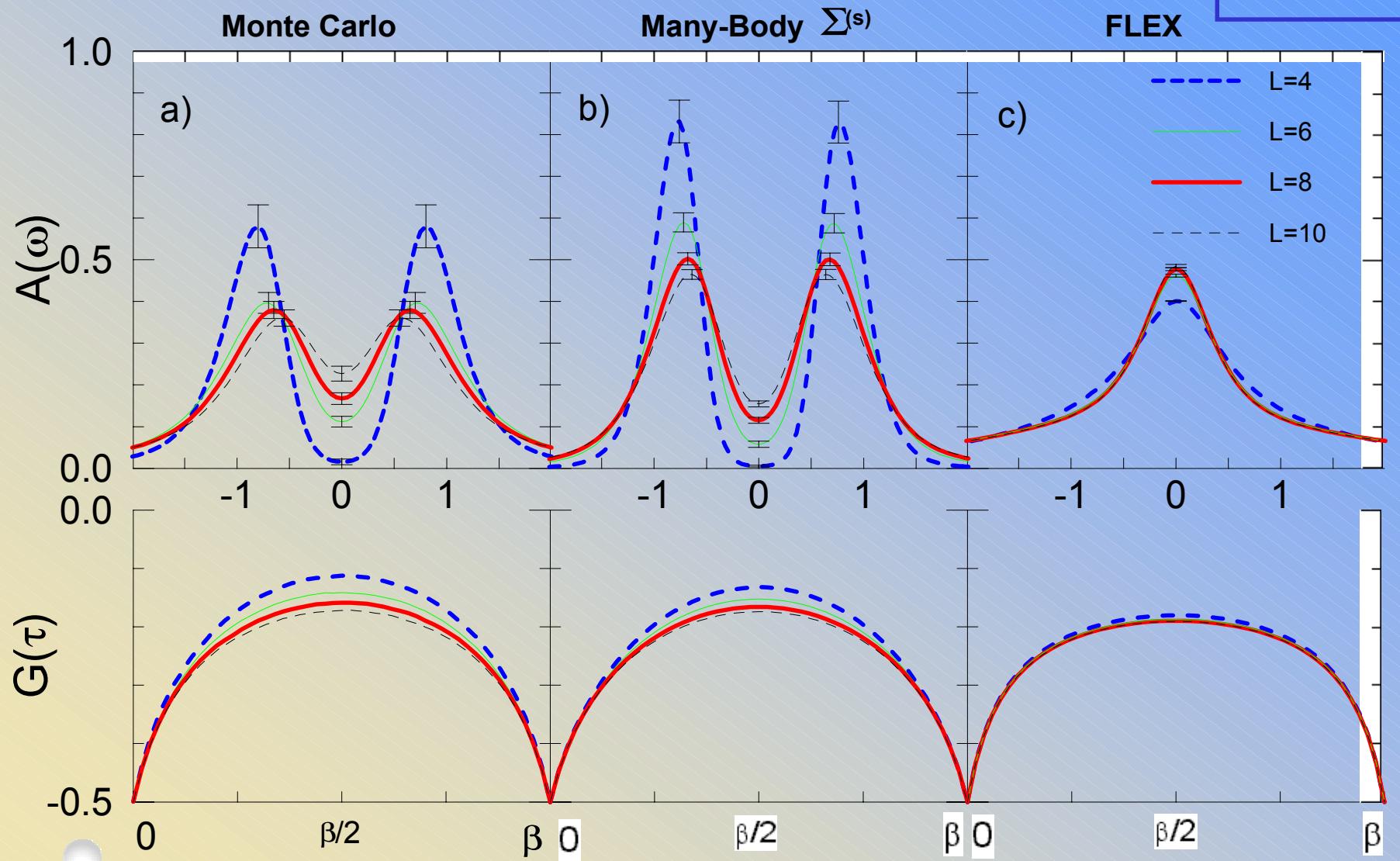
-F.L.  
parameters

-Self also  
Fermi-liquid



QMC + cal.: Vilk et al. P.R. B 49, 13267 (1994)

$U = +4$

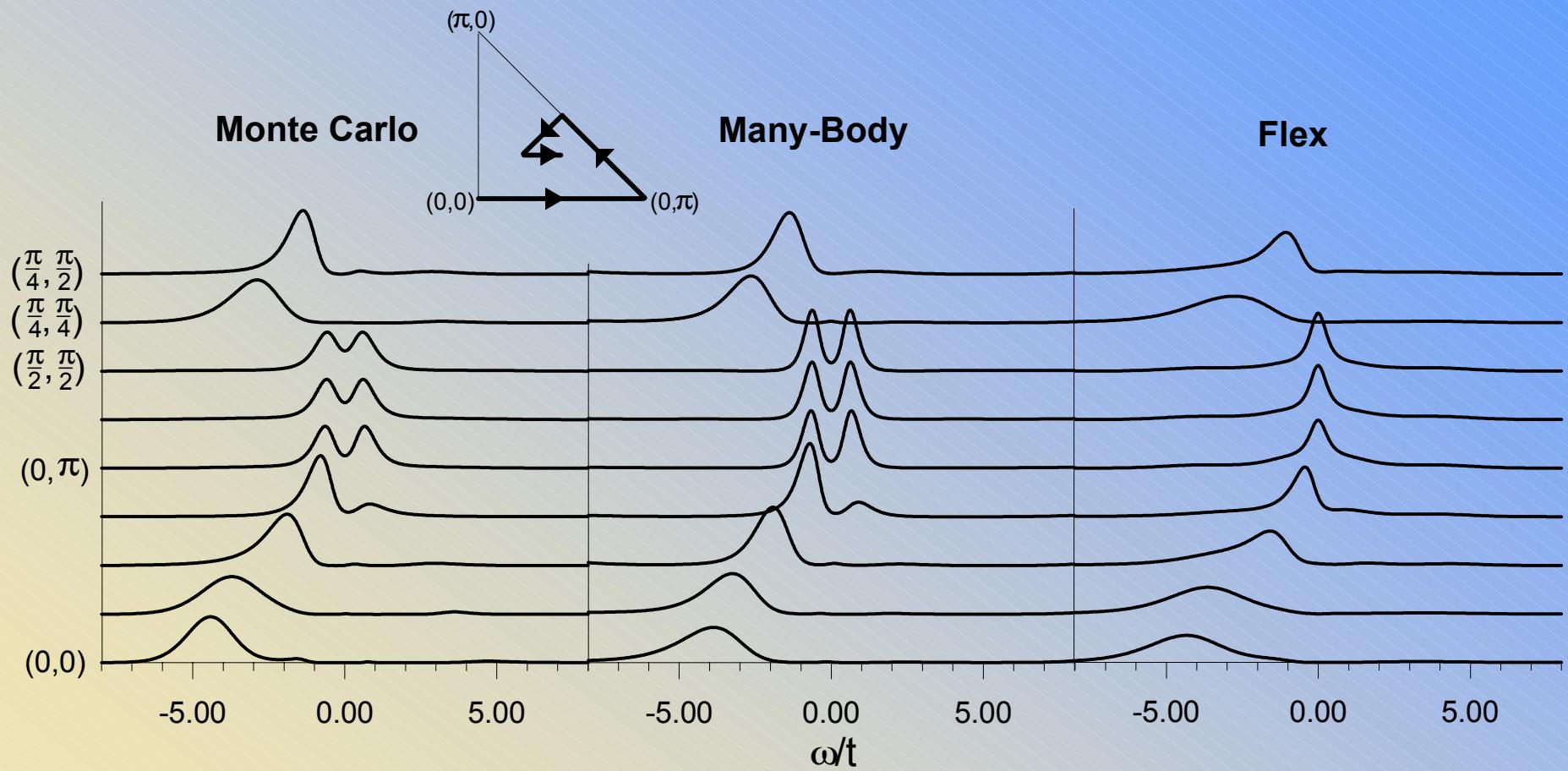


Calc. + QMC: Moukouri et al. P.R. B 61, 7887 (2000).

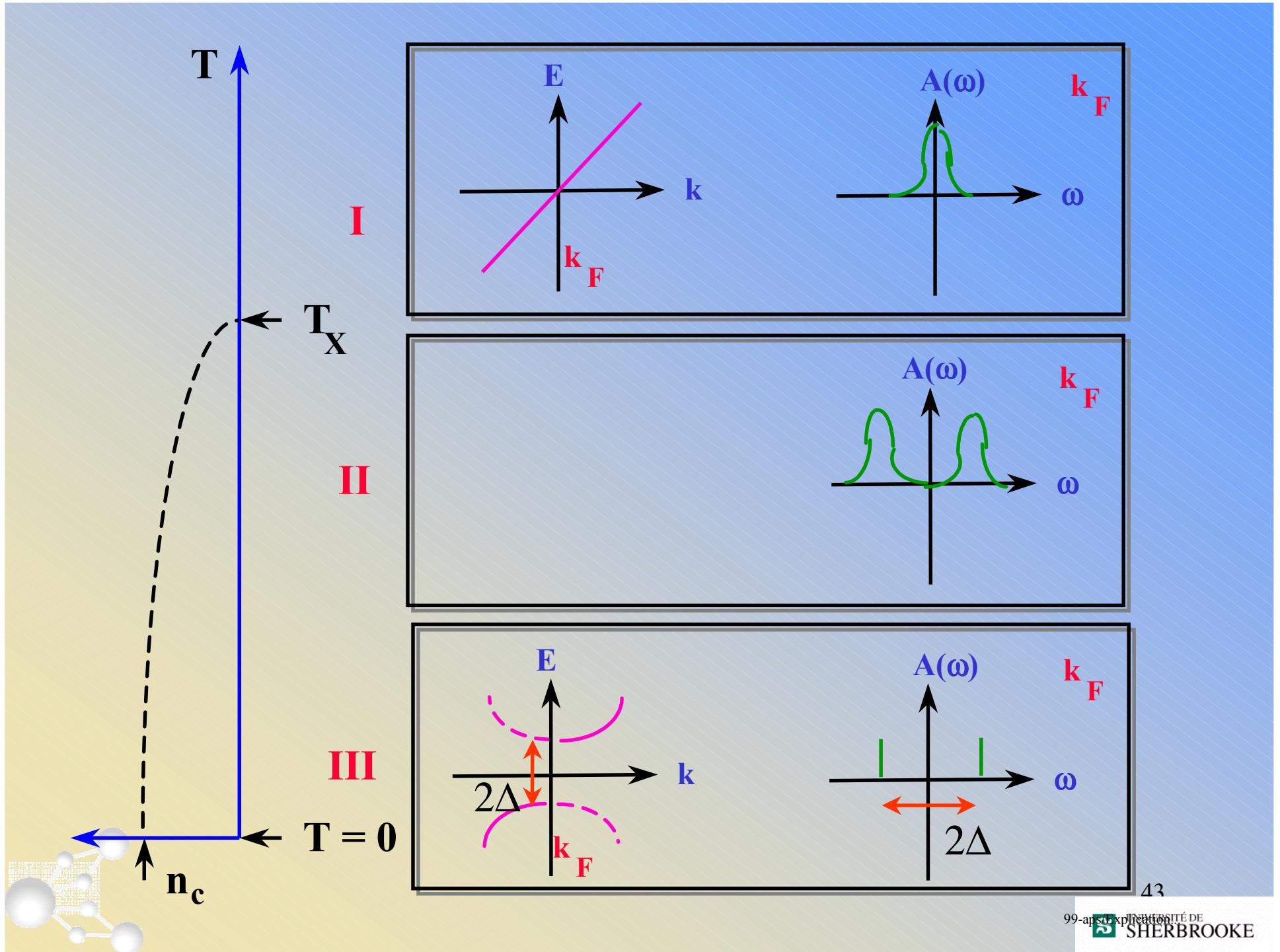


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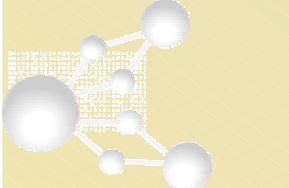
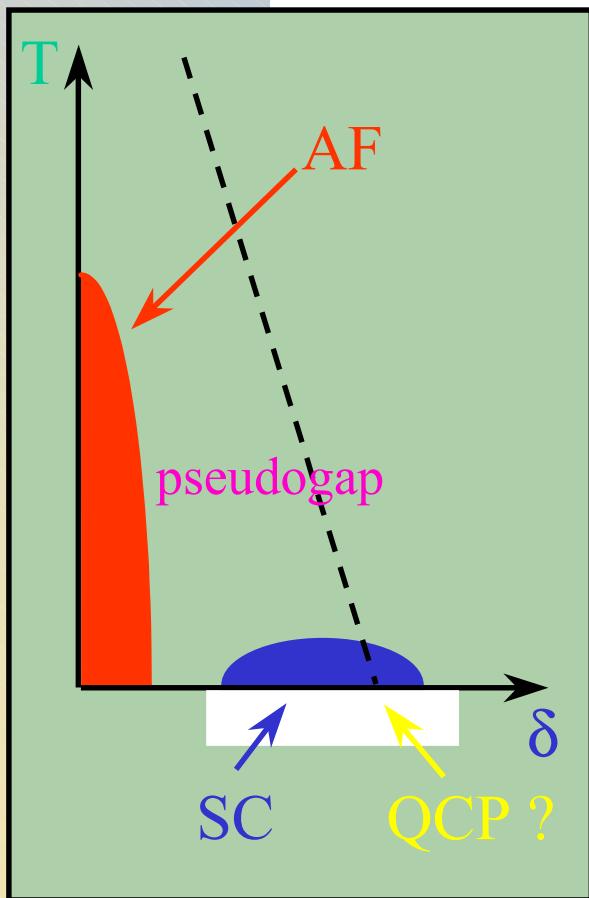
$U = +4$



Calc. + QMC: Moukouri et al. P.R. B 61, 7887 (2000).



$U > 0$



- Evidence against renormalized classical regime for spin fluctuations in pseudogap regime.

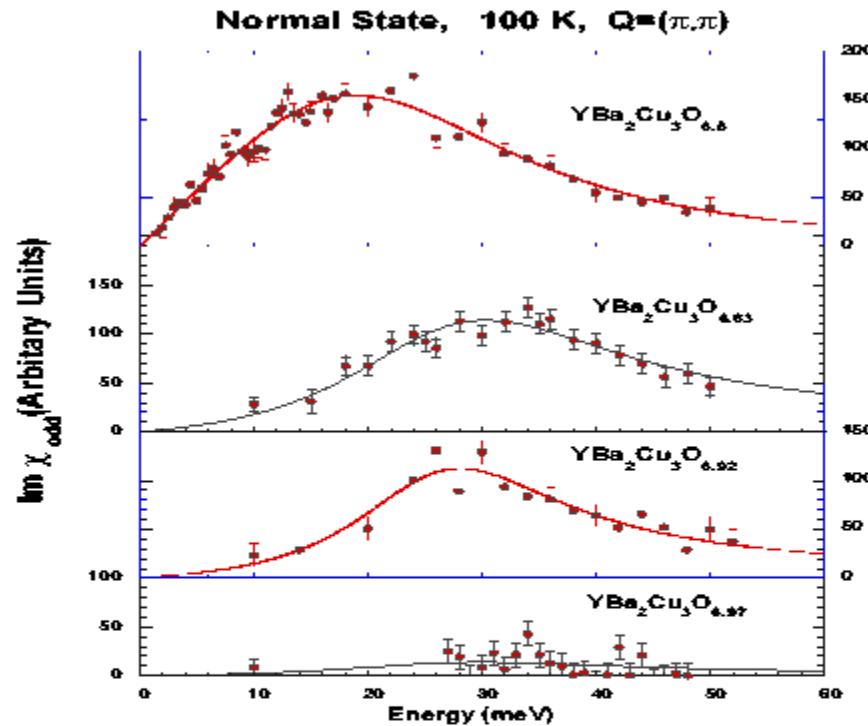


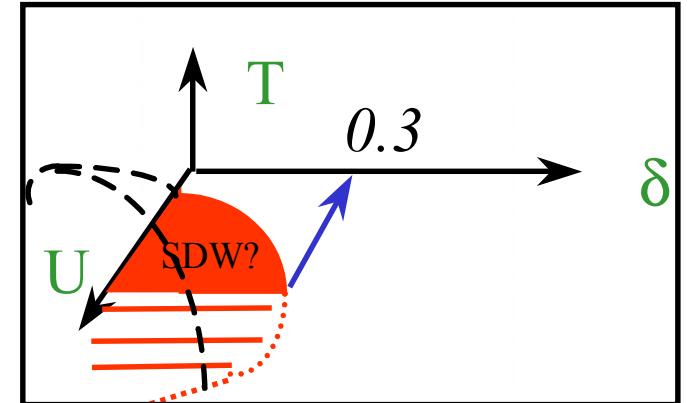
Figure 2: Normalized imaginary part of the spin susceptibility at the AF wavevector in the normal state, at  $T = 100$  K, for four oxygen contents in YBCO ( $T_c = 15, 85, 91, 92.5$  K for  $x = 0.5, 0.83, 0.92, 0.97$  respectively). These curves have been normalized to the same units using standard phonon calibration<sup>14</sup> (100 counts in the vertical scale roughly correspond to  $\sim 250 \mu_B^2/eV$  in absolute units) (from<sup>10</sup>).

Philippe Bourges cond-mat/0009373

$U > 0$

- Quantum critical point,  $d = 2$ :
  - Instability at incommensurate  $q$
  - Largest doping : 0.315

Vilk et al. P.R. B 49, 13267 (1994)

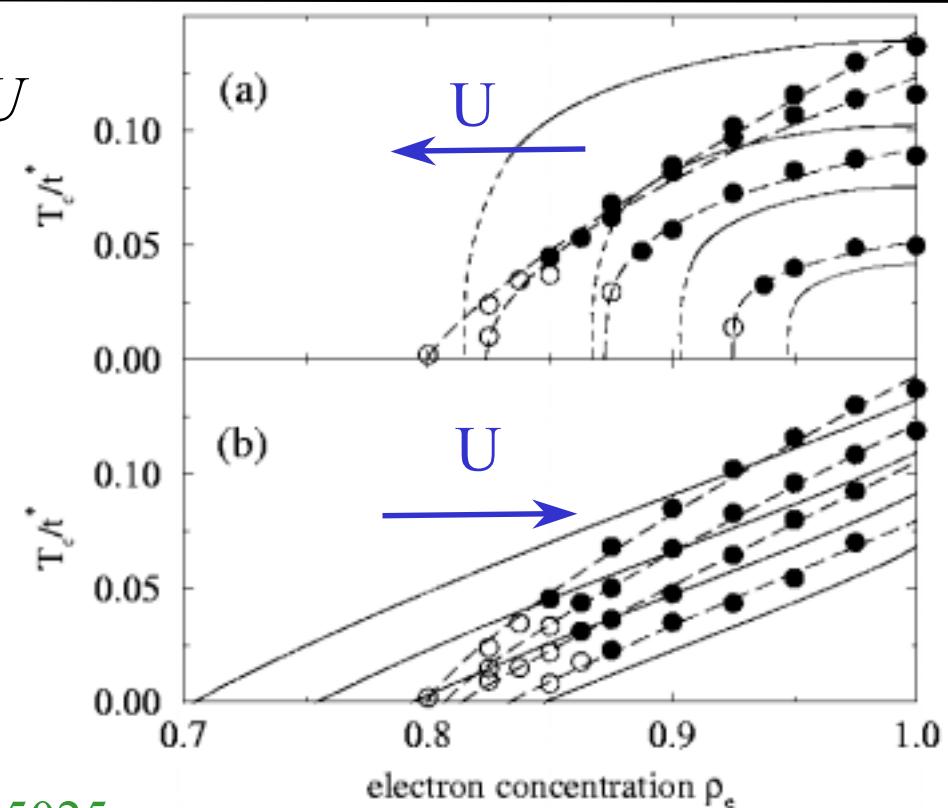


- Decreases with increasing  $U$

$$U < W$$

$d = \text{infinity}$

$$U > W$$



Freericks, Jarrell cond-mat/9405025

## Results of the analogous procedure for $U < 0$

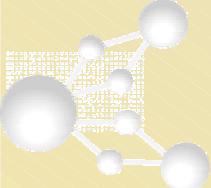
$$U_{pp} = U \frac{\langle (1 - n_\uparrow) n_\downarrow \rangle}{\langle 1 - n_\uparrow \rangle \langle n_\downarrow \rangle}. \quad (5)$$

$$\chi_p^{(1)}(q) = \frac{\chi_0^{(1)}(q)}{1 + U_{pp} \chi_0^{(1)}(q)} \quad (6)$$

$$\frac{T}{N} \sum_q \chi_p^{(1)}(q) \exp(-iqn0^-) = \langle \Delta^\dagger \Delta \rangle = \langle n_\uparrow n_\downarrow \rangle. \quad (7)$$

$$\Sigma^{(1)} \simeq \frac{U}{2} - \frac{U_{pp}(1-n)}{2} \quad (8)$$

$$\Sigma_\sigma^{(2)}(k) = Un_{-\sigma} - U \frac{T}{N} \sum_q U_{pp} \chi_p^{(1)}(q) G_{-\sigma}^{(1)}(q-k) \quad (9)$$



Satisfies Pauli principle and generalization of  $f$ -sum rule

$$\int \frac{d\omega}{\pi} \text{Im} \chi^{(1)}(\mathbf{q}, \omega) = \langle [\Delta_{\mathbf{q}}(0), \Delta_{\mathbf{q}}^\dagger(0)] \rangle = 1 - n \quad ; \quad \forall \mathbf{q} \quad (10)$$

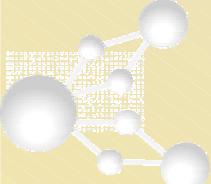
$$\int \frac{d\omega}{\pi} \omega \text{Im} \chi^{(1)}(\mathbf{q}, \omega) = \left[ \frac{1}{N} \sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} + \varepsilon_{-\mathbf{k}+\mathbf{q}}) (1 - 2 \langle n_{\mathbf{k}\uparrow} \rangle) \right] \quad (11)$$

$$- 2 \left( \mu^{(1)} - \frac{U}{2} \right) (1 - n) \quad ; \quad \forall \mathbf{q} \quad (12)$$

Internal accuracy check (For both  $U > 0$  and  $U < 0$ ).

$$\frac{1}{2} \text{Tr} [\Sigma^{(2)} G^{(1)}] = \lim_{\tau \rightarrow 0^-} \frac{T}{N} \sum_k \Sigma_\sigma^{(2)}(k) G_\sigma^{(1)}(k) e^{-ik_n \tau} = U \langle n_\uparrow n_\downarrow \rangle$$

Check :  $\text{Tr} [\Sigma^{(2)} G^{(1)}] \sim \text{Tr} [\Sigma^{(2)} G^{(2)}]$

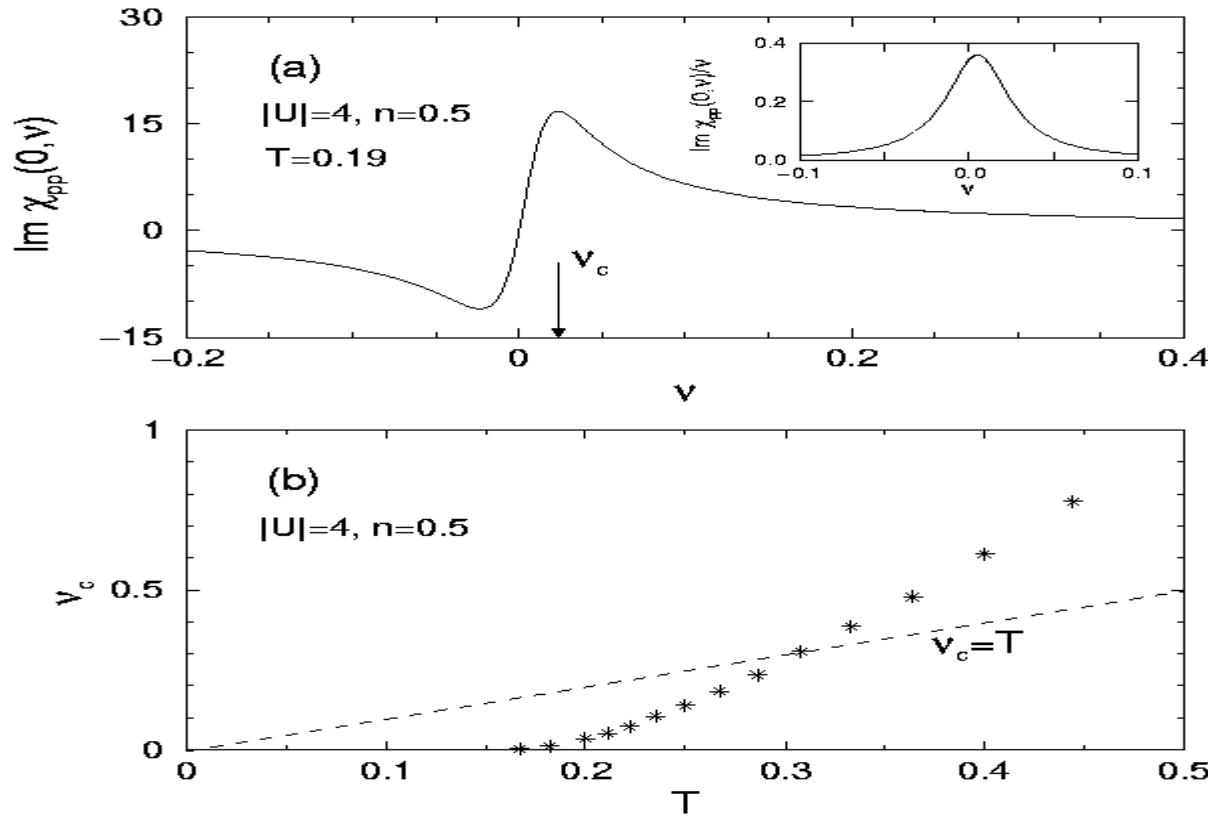


## Results:

- Mechanism for pseudogap

**U = - 4**

- (analogous to  $U > 0$ ) : Vilk *et al.* Europhys. Lett. 33, 159 (1996)  
Pines, Schmalian (98)
- Enter the renormalized-classical regime. **N.B.  $d = 2$**

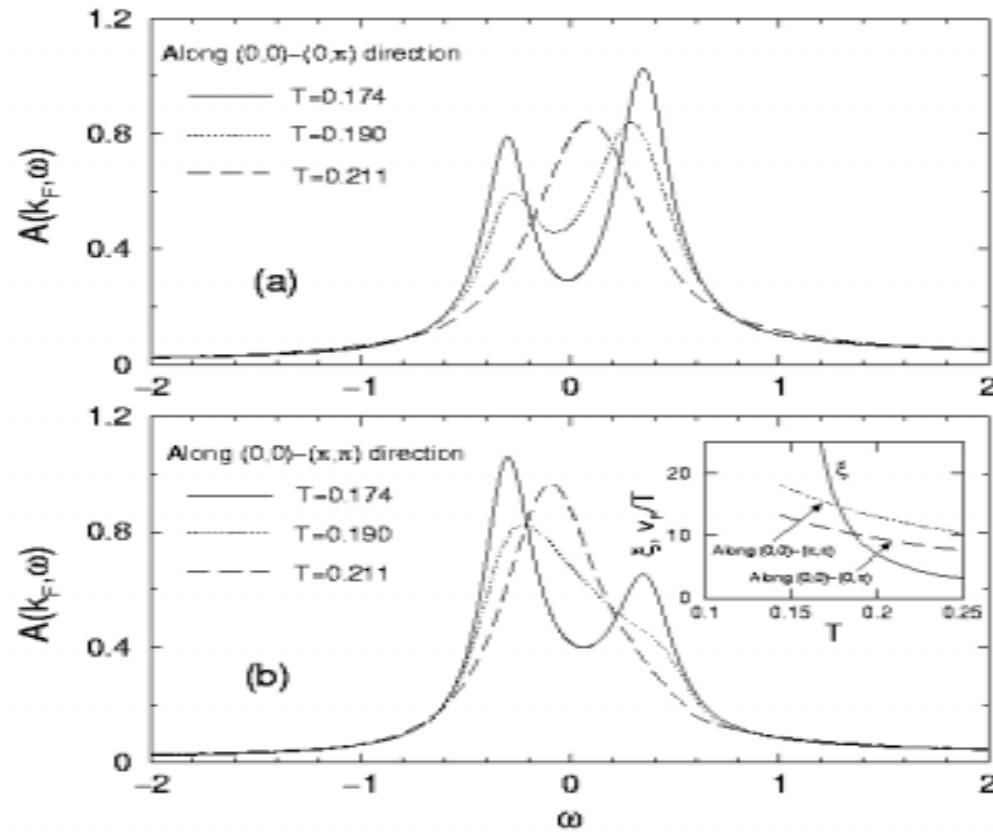


## Results:

- Mechanism for pseudogap

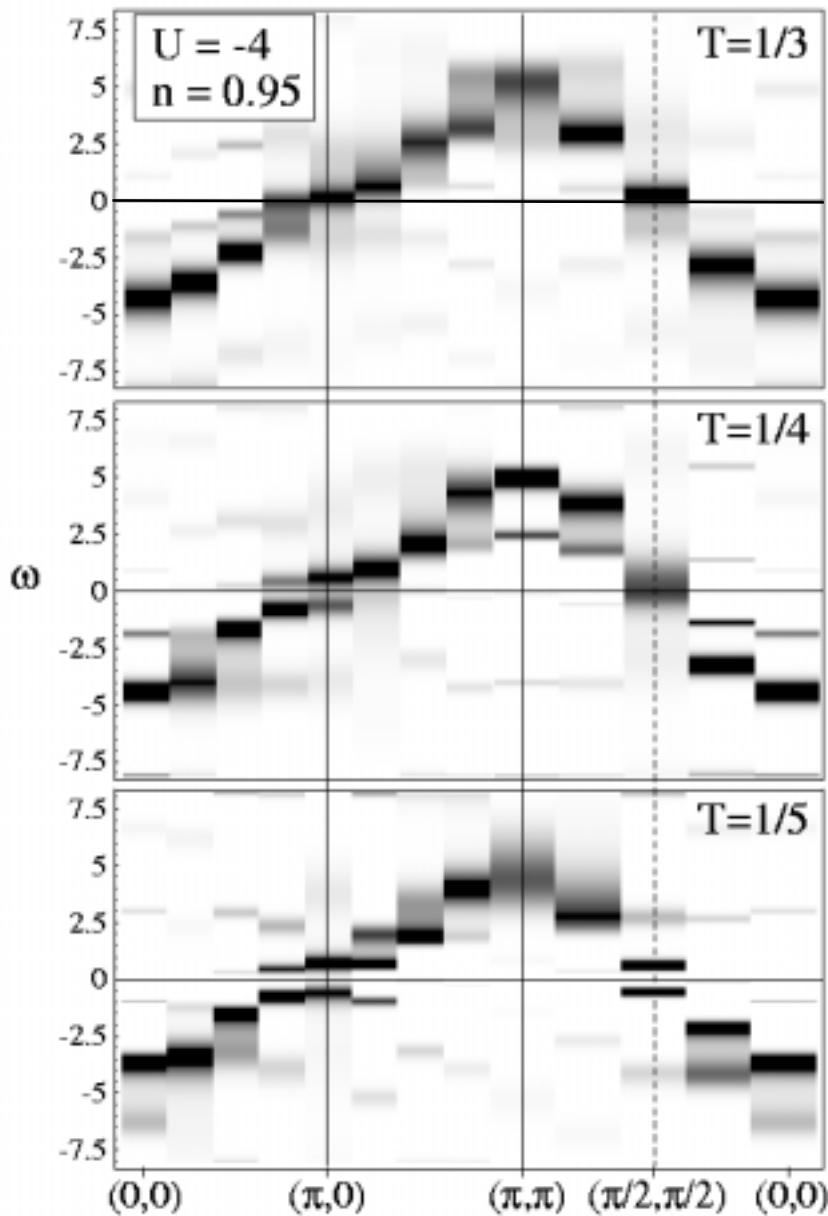
**U = - 4**

- Pairing correlation length larger than single-particle thermal de Broglie wavelength ( $v_F / T$ )



$$\xi \sim 1.3 \xi_{th}$$

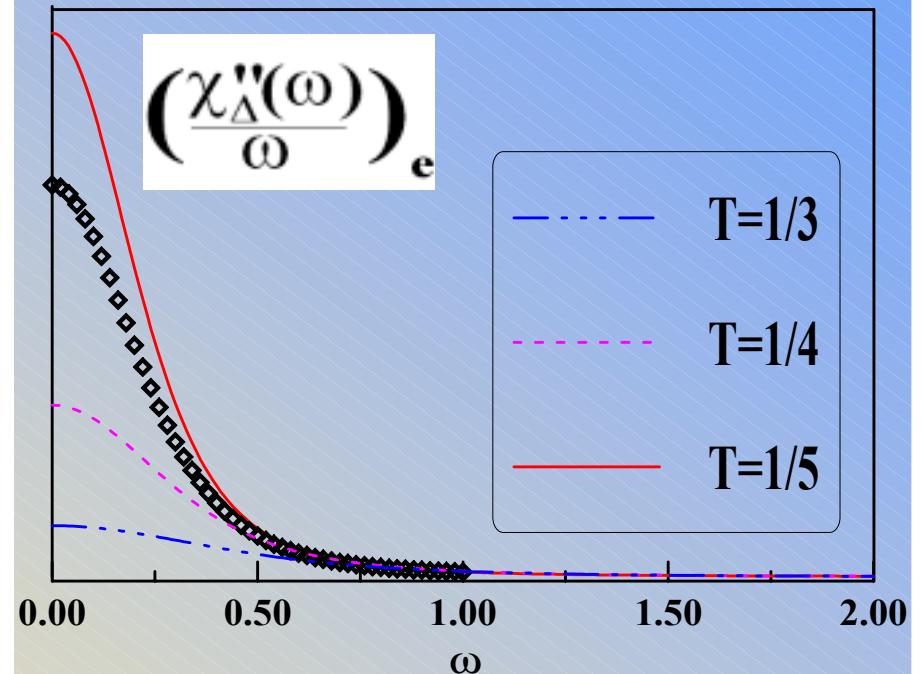
# Mechanism for pseudogap formation in the attractive model:



$U = -4$

$d = 2$  is crucial

Even part of the pair susceptibility at  $q = 0$ , for different temperatures



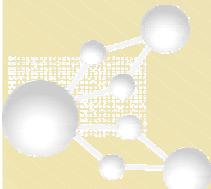
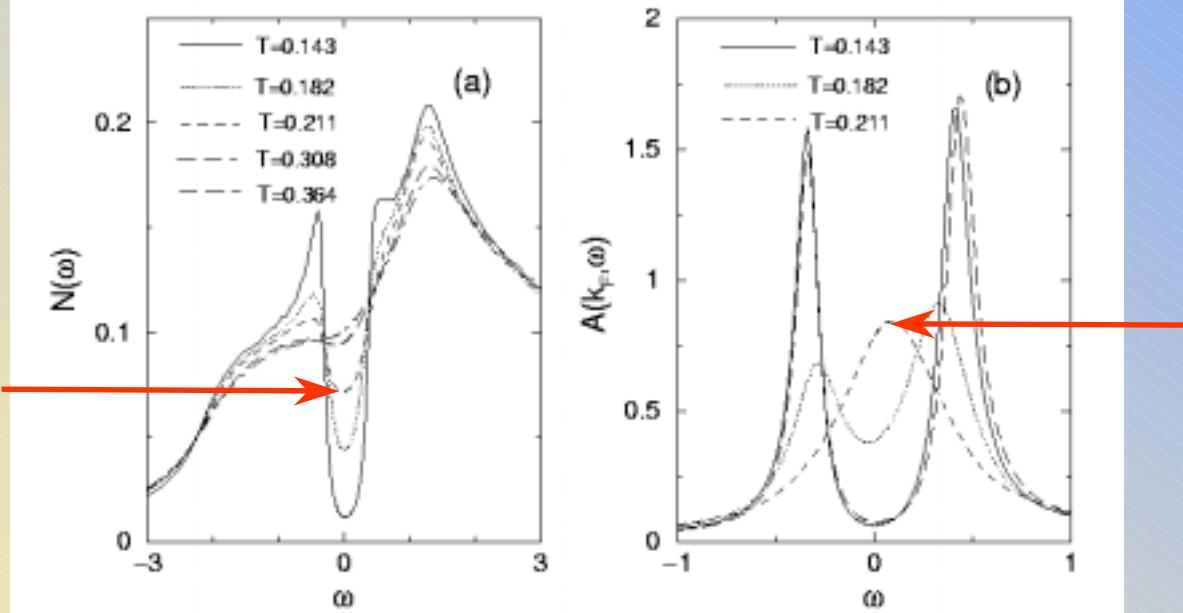
Allen, et al. P.R. L 83, 4128 (1999)

## Results:

- Spectral weight rearrangement

**U = - 4**

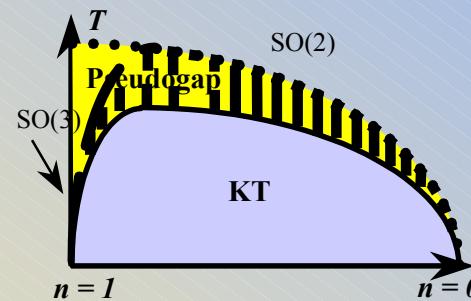
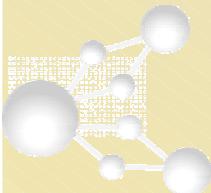
- Pseudogap appears first in total density of states
- Fills in instead of opening up
- Rearrangement over huge frequency scale compared with either  $T$  or  $\Delta T$ . ( $\Delta T \sim 0.03$ ,  $T \sim 0.2$ ,  $\Delta\omega \sim 1$ )



**U < 0**

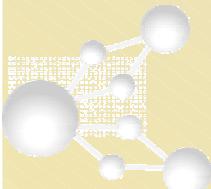
## Pairing-fluctuation induced pseudogap

- Slightly Overdoped High-Tc Superconductor  $TlSr_2CaCu_2O_{6.8}$   
Guo-qing Zheng *et al.*, P. R. L. **85**, 405 (2000)
  - Pseudogap in Knight shift and NMR relaxation strongly  $H$  dependent, contrary to underdoped (up to 23 T).
- Underdoped in a range  $\Delta T \sim 15 K$  near  $T_c$  see evidence for renormalized classical regime ( $KT$  behavior).  
Corson *et al.* Nature, **398**, 221 (1999).
- Higher symmetry group creates large range of  $T$  where there is a pseudogap.  
Allen et al. P.R.L. **83**, 4128 (1999)



## (d) Approaching from strong coupling

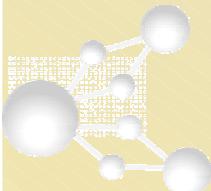
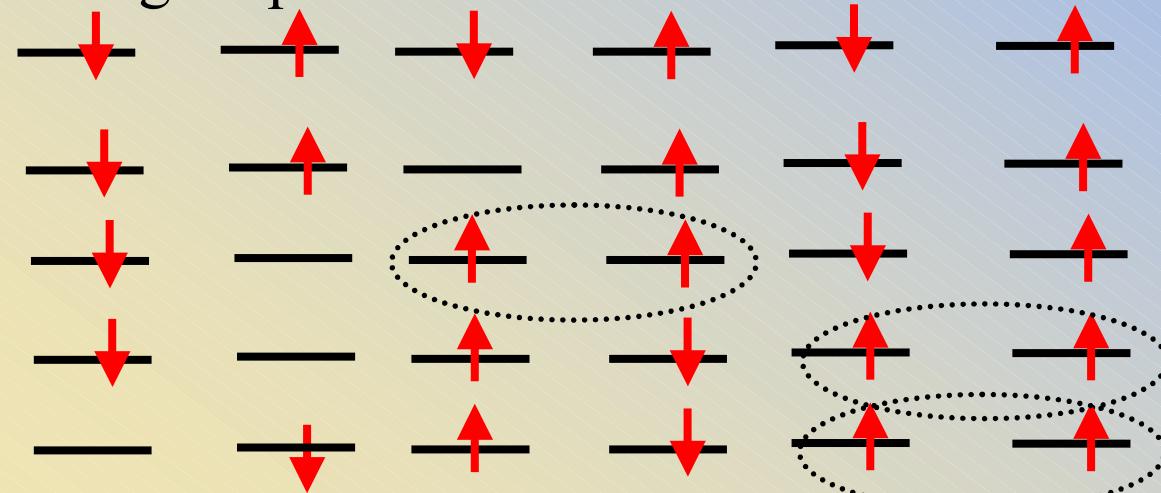
- Straight perturbative treatment is difficult:
  - No Wick's theorem
  - Non-causality
    - (S. Pairault, D. Senechal, A.-M. S. T. Eur. Phys. J. B 16, 85 (2000))
- Slave bosons and slave fermions
  - $c_{\uparrow}^+ (1 - n_{\downarrow}) \rightarrow f_{\uparrow} b^+$  or  $f b_{\uparrow}^+$
  - Constraint:  $(\sum_{\sigma} f_{\sigma}^+ f_{\sigma}) + b^+ b = 1$ 
    - Mean field : constraint with Lagrange multiplier
- Gauge theory
  - $f_{\uparrow}^+ \rightarrow e^{i\theta} f_{\uparrow}^+$
  - $b \rightarrow e^{-i\theta} b$
  - $\lambda \rightarrow \lambda + \delta \theta / \delta \tau$



- Gauge theory:
  - Break Gauge symmetry
  - Look for mass of gauge field (stable if mass)
  - Find topological excitations
    - (charge carriers prop. to  $\delta$ )
- There are ambiguities (Slave-fermions vs slave-bosons)
  - e.g. limit  $J = 0$  (Nagaoka)
 

*(Daniel Boies, F. Jackson and A.-M.S. T. Int. J. Mod. Phys. **9**, 1001 (1995))*

- Spin-charge separation



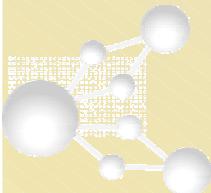
## (e) Phonons

Susceptibilité de paires de type  $d_{x^2-y^2}$  pour un réseau 6x6

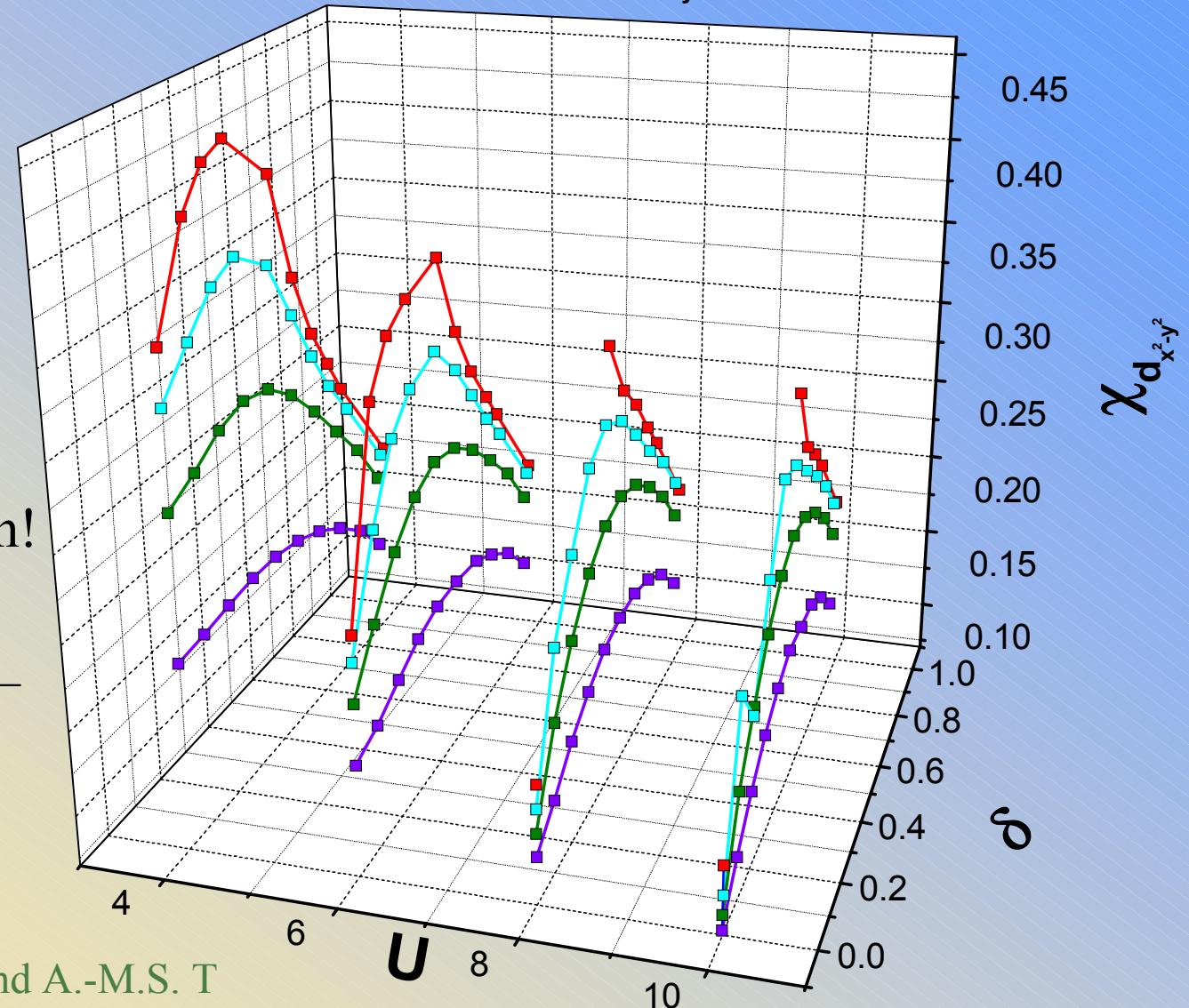
$\beta = 1$   
 $\beta = 2$   
 $\beta = 3$   
 $\beta = 4$

Would need oxygen!

$$\chi_d' = \frac{\chi_d}{1 - V\chi_d}$$

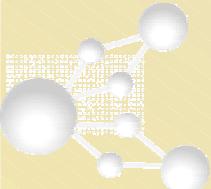


J.-S. Landry and A.-M.S. T

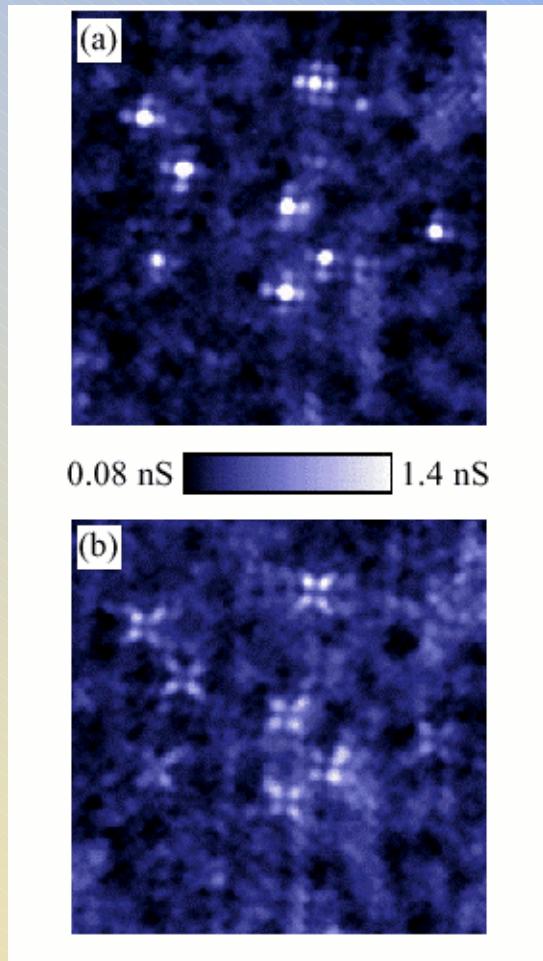


## (f) Inhomogeneities

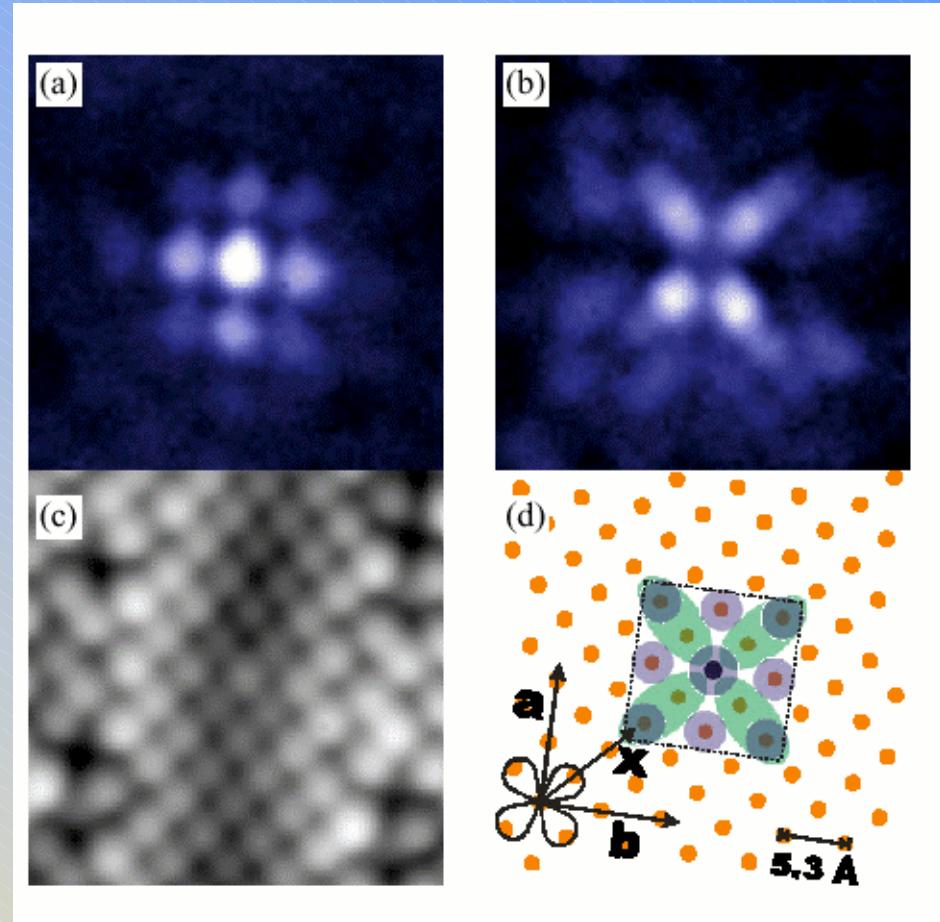
- Disorder is important in underdoped
  - Even without interactions, complicated: localization
  - Main theoretical tools: impurity averaging, Replica trick
- Instabilities to inhomogeneous ground states:
  - $S_Q \cdot S_{Q-2Q}$ 
    - $n = 1$  magnetic « anti-stripes »
    - $n = 0.5$  « charged stripes »
- Using impurities as a « diagnostic tool » for superconducting state
  - Complicated : Kondo effect etc...



## Inhomogeneities



## Impurities



E.W. Hudson *et al.* cond-mat/0104237

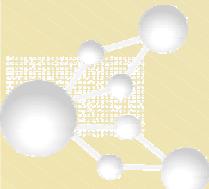
See also : C. Howald, P. Fournier, A. Kapitulnik cond-mat/0101251

(g) Quantum critical points....

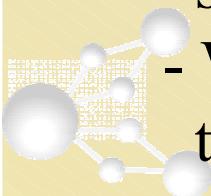


## V. Conclusion

- The pseudogap summarizes anomalous « normal state» properties
  - It is ill-understood
  - Need to understand it to understand the phase diagram and superconductivity.
- It is the motivation for a vast body of work in many directions
- Methods have to be developed at the same time
- Sociology

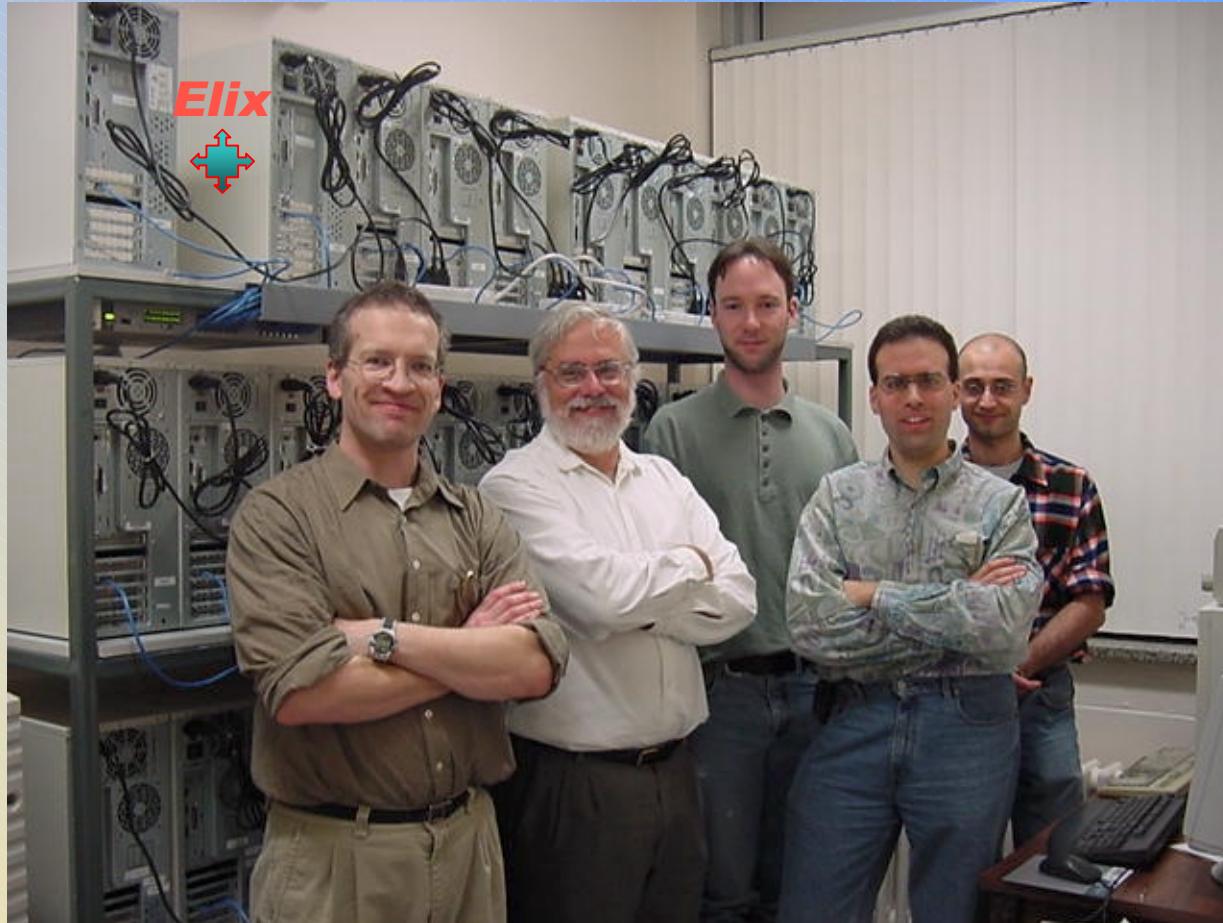


- How can we understand electronic systems that show both localized and extended character?
- Why do both organic and high-temperature superconductors show broken-symmetry states where mean-field-like quasiparticles seem to reappear?
- Why is the condensate fraction in this case smaller than what would be expected from the shape of the would-be Fermi surface in the normal state?
- Are there new elementary excitations that could summarize and explain in a simple way the anomalous properties of these systems?
- Do quantum critical points play an important role in the Physics of these systems?
- Are there new types of broken symmetries?
- How do we build a theoretical approach that can include both strong-coupling and  $d = 2$  fluctuation effects?
- What is the origin of d-wave superconductivity in the high-temperature superconductors?



Michel Barrette

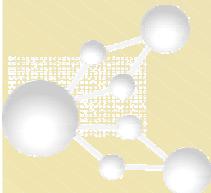
Mehdi Bozzo-Rey



David Sénéchal

A.-M.T.

Alain Veilleux





Steve Allen



François Lemay



David Poulin

Liang Chen



Yury Vilk



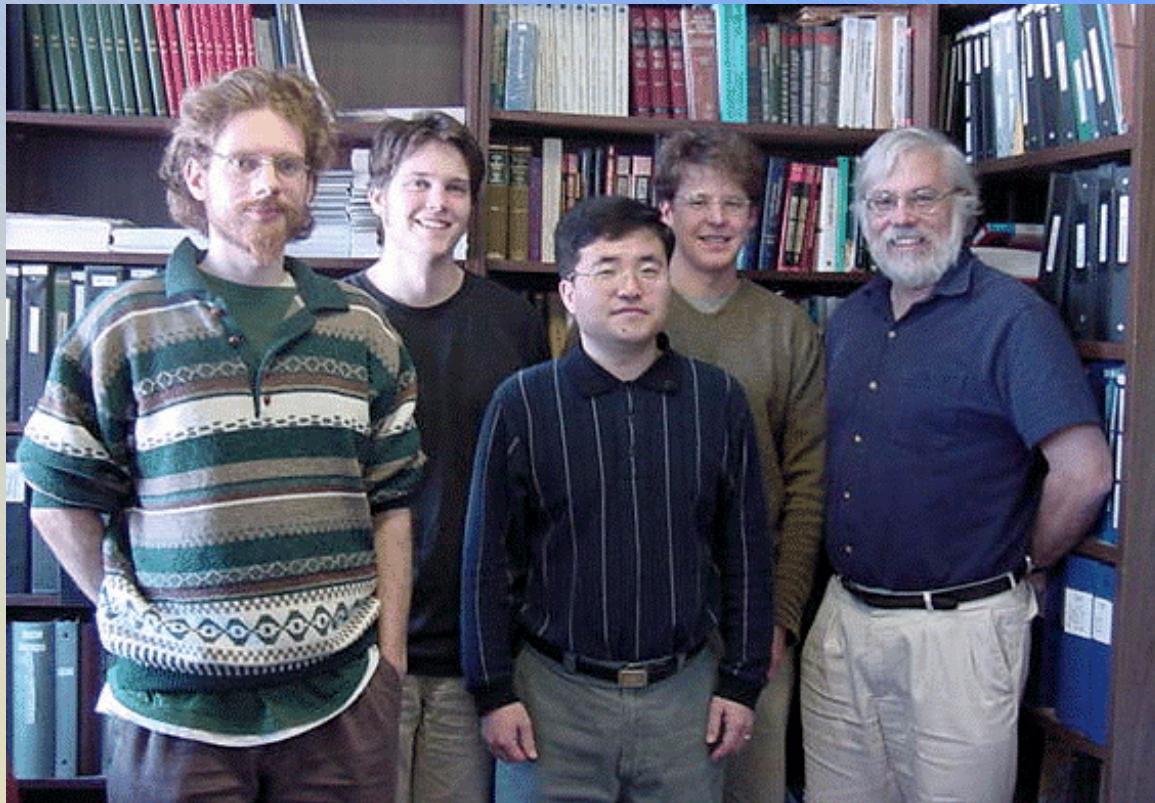
Samuel Moukouri



Hugo Touchette



Sébastien Roy    Alexandre Blais



Jean-Sébastien Landry

A-M.T.

Bumsoo Kyung

