

Low-energy quasiparticles in cuprate superconductors: A quantitative analysis

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A residual linear term is observed in the thermal conductivity of optimally doped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ at very low temperatures whose magnitude is in excellent agreement with the value expected from Fermi-liquid theory and the d -wave energy spectrum measured by photoemission spectroscopy with no adjustable parameters. This solid basis allows us to make a quantitative analysis of thermodynamic properties at low temperature and establish that thermally excited quasiparticles are a significant, perhaps even the dominant, mechanism in suppressing the superfluid density in cuprate superconductors $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ and $\text{YBa}_2\text{Cu}_3\text{O}_7$.

The superconducting order parameter of the archetypal high- T_c compounds $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ is widely agreed to have d -wave symmetry, yet there is no consensus on the correct theoretical description of their superconducting state properties, let alone those of the metallic state. A fundamental issue in the current debate is the nature of the electronic excitations in these systems, and whether long-lived quasiparticles exist¹ or not.² Another debate concerns the dominant mechanism responsible for the thermal suppression of the superfluid density, whether it be d -wave nodal quasiparticles³ or phase fluctuations,⁴ for example. One way to shed light on these issues is to go beyond the usual qualitative temperature dependence of physical properties, and look closely at their magnitude. Our specific approach is to examine quantitatively the basic thermodynamic and transport properties of these two superconductors within a Fermi-liquid description of d -wave quasiparticles grounded in a spectroscopic measurement of the energy spectrum, and see whether a consistent description at low energies can be achieved.

The $d_{x^2-y^2}$ gap function goes to zero at four nodes along the $k_x = \pm k_y$ directions, producing a conelike quasiparticle excitation spectrum at low energies:

$$E = \hbar \sqrt{v_F^2 k_1^2 + v_2^2 k_2^2}, \quad (1)$$

where v_F and v_2 are the energy dispersions, or quasiparticle velocities, along directions normal ($\parallel \mathbf{k}_1$) and tangential ($\parallel \mathbf{k}_2$) to the Fermi surface at each node. This spectrum is associated with the two-dimensional CuO_2 plane that is the fundamental building block of all cuprates. It neglects any possible dispersion in the third direction (along the c axis), as well as excitations associated with the one-dimensional CuO chains found in some crystal structures, notably in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (along the b axis).

This simple spectrum gives rise to a quasiparticle density of states which is linear in energy:

$$N(E) = \frac{2}{\pi \hbar^2} \frac{1}{v_F v_2} E, \quad (2)$$

which in turn leads to a T^2 dependence of the electronic specific heat and a linear T dependence of the superfluid density, for example. In a realistic treatment, one needs to include the effect of impurity scattering and electron-electron interactions. One usually accounts for the former in terms of a single, isotropic scattering rate, parametrized by an impurity bandwidth γ . At energies below γ , known as the ‘‘dirty’’ limit, one expects a profound modification of the density of states, which acquires a residual finite value $N(0) \propto \gamma$. At energies well above γ , in the ‘‘clean’’ limit, $N(E) \propto E$ and one recovers many of the straightforward temperature dependences. Going beyond this, Durst and Lee recently included vertex corrections, which arise because of the anisotropy of scattering in a d -wave superconductor.⁵ The importance of Fermi-liquid corrections has also been emphasized, whereby electron-electron interactions renormalize the normal fluid density.^{3,5,6}

In this paper, we use a measurement of the in-plane thermal conductivity at very low temperatures to extract a value for the ratio v_F/v_2 in optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO) and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (BSCCO). We proceed to show first that for BSCCO this ratio is in excellent agreement with the values of v_F and v_2 measured separately by angle-resolved photoemission spectroscopy (ARPES). We then use the ratio to calculate the drop in superfluid density within a Fermi-liquid description and compare this with the experimental results obtained from penetration depth measurements. Finally, we extend our quantitative analysis to include specific heat measurements in YBCO. From the overall analysis, we conclude that the superconducting state of the cuprates is well described by Fermi-liquid theory, at least at low energy and optimal doping.

Thermal conductivity

The thermal conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ was measured using a steady-state method

TABLE I. Comparison of $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ at optimal doping. In YBCO, all directional properties are for the a axis (no chain contribution). d/n is the average separation between CuO_2 planes stacked along the c axis. The zero temperature penetration depth $\lambda(0)$ was measured by far-infrared reflectivity in YBCO (Ref. 22) and by dc magnetization in BSCCO (Ref. 23). The Fermi velocity v_F and momentum k_F were obtained from angle-resolved photoemission (Refs. 11 and 13). v_F/v_2 is the ratio of quasiparticle velocities, obtained via Eq. (3), using the residual linear term κ_0/T measured in the thermal conductivity at $T \rightarrow 0$. $S = d\Delta(\phi)/d\phi = \hbar k_F v_2$ is the slope of the gap at the node calculated using k_F , v_F and v_F/v_2 . Δ_{\max} is the gap maximum as seen in c -axis tunneling by scanning tunnel microscopy (Refs. 14–16) with $S = \mu \Delta_{\max}$. The linear drop in superfluid density with temperature is expressed as $\lambda^2(0)d\lambda^{-2}/dT$, obtained from the penetration depth measured at microwave frequencies (Refs. 20 and 21). α^2 is the Fermi-liquid correction computed from Eq. (7), using the measured values of $d\lambda^{-2}/dT$ and κ_0/T .

	$\frac{d}{n}$	T_c	$\lambda(0)$	v_F	k_F	$\frac{\kappa_0}{T}$	$\frac{v_F}{v_2}$	S	Δ_{\max}	μ	$\lambda^2(0)\frac{d\lambda^{-2}}{dT}$	α^2
	(Å)	(K)	(Å)	(km/s)	(Å ⁻¹)	($\frac{\text{mW}}{\text{K}^2\text{cm}}$)		(meV)		(meV)	(K ⁻¹)	
YBCO	5.85	93.6	1600	~250	~0.8	0.14	14	94	~20	4.7	(205 K) ⁻¹	0.46
BSCCO	7.72	89	2100	250	0.74	0.15	19	64	~40	1.6	(120 K) ⁻¹	0.43

described elsewhere.⁷ The samples were single crystals grown via standard flux techniques and oxygenated so as to obtain the maximum T_c (optimal doping), quoted in Table I. Both crystal structures are made of CuO_2 planes stacked along the c axis, with a density 30% higher in YBCO, due to its lower average interplane spacing, given in Table I. The conductivity of YBCO was measured along the a axis in untwinned crystals so as to avoid the contribution of CuO chains.

In Fig. 1 we present the low-temperature thermal conductivity κ of BSCCO, and compare it with that of YBCO obtained previously.⁸ By plotting κ/T vs T^2 , we can separate the linear quasiparticle term from the cubic phonon term (see Ref. 7). A finite residual linear term κ_0/T (the value of κ/T as $T \rightarrow 0$) is observed of similar magnitude for the two cuprates given in Table I. (Note that the value for YBCO is an average over several samples,⁸ only one of which is displayed in Fig. 1.) The error bar on these numbers is approxi-

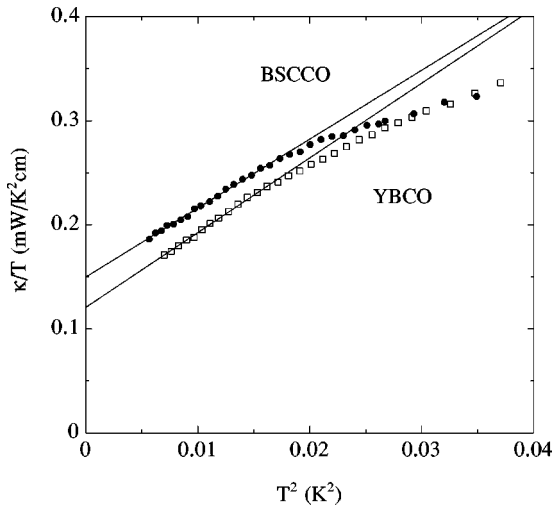


FIG. 1. Thermal conductivity divided by temperature vs T^2 of $\text{YBa}_2\text{Cu}_3\text{O}_7$ (squares) and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (circles), at optimum doping. The lines are linear fits to the data below 130 mK, with extrapolated values given in Table I.

mately $\pm 20\%$ arising about equally from the uncertainty in the extrapolation and in the geometric factor of each sample.

Calculations for the transport of heat by d -wave quasiparticles in two dimensions give^{5,9}

$$\frac{\kappa_0}{T} = \frac{k_B^2}{3\hbar} \frac{n}{d} \left(\frac{v_F}{v_2} + \frac{v_2}{v_F} \right) \approx \frac{k_B^2}{3\hbar} \frac{n}{d} \left(\frac{v_F}{v_2} \right), \quad (3)$$

where n/d is the stacking density of CuO_2 planes. The residual conduction is due to a fluid of zero-energy quasiparticles induced by the pair-breaking effect of impurity scattering near the nodes in the gap and it is independent of impurity concentration. This universal character of κ_0/T was demonstrated explicitly for both YBCO (Ref. 7) and BSCCO.¹⁰

Durst and Lee recently showed Eq. (3) to be valid even when vertex and Fermi-liquid corrections are taken into account,⁵ so that unlike charge transport, heat transport is *not* renormalized by either correction. The universal character and the absence of renormalization make thermal conductivity a privileged probe of the quasiparticle spectrum in a d -wave superconductor, providing a simple and direct measurement of v_F/v_2 in the cuprates. From the measured κ_0/T and the known values of n/d (see Table I), we obtain

$$\frac{v_F}{v_2} = 19 \quad \text{for BSCCO}, \quad (4)$$

$$\frac{v_F}{v_2} = 14 \quad \text{for YBCO}, \quad (5)$$

with an uncertainty of about $\pm 20\%$. (Note that the ratio for YBCO is twice the value of ~ 7 often used in the literature.)

ARPES

Angle-resolved photoemission spectroscopy has established the existence of a Fermi surface in YBCO (Ref. 11) and BSCCO (Refs. 1 and 2) and revealed directly the \mathbf{k} dependence of the gap characteristic of $d_{x^2-y^2}$ symmetry.^{12,13} In BSCCO, the nodes are along the $(0, 0)$ to (π, π) direction,

at $k=k_F=0.74 \text{ \AA}^{-1}$,¹³ where the energy has a dispersion along \mathbf{k}_1 given by $v_F=2.5 \times 10^7 \text{ cm/s}$.¹³ As for the dispersion along \mathbf{k}_2 (or ϕ), Mesot *et al.*¹³ were recently able to extract $S=|d\Delta/d\phi|_{\text{node}}$, the slope of the gap at the node. For a crystal near optimal doping ($T_c=87 \text{ K}$), they obtain $S=60 \pm 5 \text{ meV}$ ($=1.7 \Delta_{\text{max}}$, where Δ_{max} is the gap maximum at $\phi=0$). This yields $v_2=S/\hbar k_F=1.2 \times 10^6 \text{ cm/s}$, so that $v_F/v_2=20$.

A hotly debated question is whether the excitations in the vicinity of the Fermi surface, in particular along the diagonals, can be treated as the usual Landau/BCS quasiparticles.^{1,2} The excellent quantitative agreement we find between the spectroscopic and the transport measurement of v_F/v_2 in BSCCO strongly validates a Fermi-liquid description of the superconducting state in cuprates, at least at low energies.

Unfortunately, ARPES measurements in YBCO have been less successful so far. The Fermi surface is more complicated with bilayer splitting of the plane bands¹¹ and an added band for the CuO chains. When averaged over the two plane bands, the band crossing and dispersion at the Fermi energy along the $(0, 0)$ to (π, π) direction are close to those quoted above for BSCCO, namely, $v_F \approx 2.5 \times 10^7 \text{ cm/s}$ and $k_F \approx 0.8 \text{ \AA}^{-1}$,¹¹ albeit with greater uncertainty. The gap structure has not yet been resolved with sufficient resolution to provide a measurement of v_2 . The thermal conductivity data may be used instead: with $v_F \approx 2.5 \times 10^7 \text{ cm/s}$, Eq. (5) yields $v_2 \approx 1.8 \times 10^6 \text{ cm/s}$. This implies that the slope of the gap at the node in YBCO is 1.5 times *larger* than in BSCCO, with $S=\hbar k_F v_2 \approx 95 \text{ meV}$, in contrast with evidence from STM measurements of *c*-axis tunneling that the gap *maximum* in YBCO is *smaller* than in BSCCO, namely, $\Delta_{\text{max}} \approx 20 \text{ meV}$ (Refs. 14 and 15) vs 40 meV .¹⁶ This suggests a strikingly different angular dependence of the gap function with a ratio of slope to gap maximum 3 times larger in YBCO (see Table I) under the assumption that v_F is the same in both materials.

Superfluid density

As the temperature is increased from $T=0$, the thermal excitation of nodal quasiparticles causes the normal fluid density $\rho_n(T)$ to grow linearly with temperature. In the clean limit at low temperature,^{3,5,6,17}

$$\frac{\rho_n(T)}{m} = \frac{2 \ln 2}{\pi} \frac{k_B}{\hbar^2} \frac{n}{d} \alpha^2 \left(\frac{v_F}{v_2} \right) T, \quad (6)$$

where m is the mass of the carriers and α^2 is the Fermi-liquid correction for charge currents.¹⁸

A linear temperature dependence of $\rho_n(T)$ is a characteristic feature of most cuprate superconductors, as revealed through measurements of the penetration depth $\lambda(T)$ via the relation $\rho_s(T)/m = \rho_s(0)/m - \rho_n(T)/m = c^2/4\pi e^2 \lambda^2(T)$. From the data of Hardy and co-workers on untwinned crystals of YBCO (Refs. 19 and 20)—again taking the *a*-axis results to avoid chain contributions—and from measurements by Waldram and co-workers in BSCCO²¹ one finds the slope of $\lambda^2(0)/\lambda^2(T)$ at low temperatures given in Table I for optimal doping. Combining Eqs. (3) and (6), we can then solve for the Fermi-liquid correction via

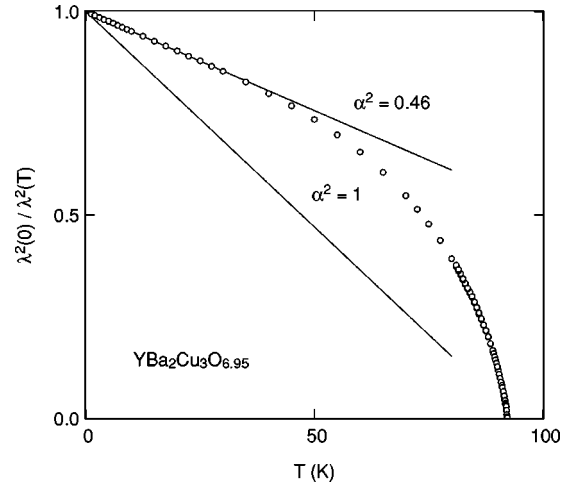


FIG. 2. Temperature dependence of the superfluid density, normalized to unity at $T=0$, for optimally doped YBCO (*a* axis) (from Ref. 20). The lines are the expected low-temperature behavior calculated from Eq. (7) with ($\alpha^2 \neq 1$) and without ($\alpha^2 = 1$) Fermi-liquid interactions.

$$\frac{d\lambda^{-2}(T)}{dT} = -2.93 \times 10^{13} \frac{\kappa_0}{T} \alpha^2, \quad (7)$$

with λ in meters and κ_0/T in $\text{W K}^{-2} \text{ m}^{-1}$. Using the values for $\lambda(0)$ quoted in Table I, we get

$$\alpha^2 = 0.43 \quad \text{for BSCCO}, \quad (8)$$

$$\alpha^2 = 0.46 \quad \text{for YBCO}. \quad (9)$$

In other words, the observed drop in superfluid density is about 2 times weaker than expected from a calculation neglecting interactions, as shown graphically for YBCO in Fig. 2. Since electrons in cuprates are highly correlated, a renormalization by a factor of 2 seems entirely plausible. The fact that it is comparable in the two compounds is not unexpected, given that the Fermi velocities, themselves renormalized by interactions,⁶ are comparable. We stress that an estimate of the Fermi-liquid correction to $\rho_s(T)$ does not require a separate knowledge of v_F and v_2 , and heat conduction—unlike heat capacity, for example—can provide directly the appropriate combination of the two parameters, i.e., their ratio. Note that in the case of BSCCO, Mesot *et al.*¹³ were the first to report an estimate of the renormalization factor, based on their ARPES data. (The fact that they obtain a slightly different value, namely, $\alpha^2 = 0.32$, is due to their use of different penetration depth data that we consider to be less reliable because they are restricted to temperatures above 17 K.)

Given the numbers that emerge from the analysis, it seems fair to conclude that the thermal excitation of quasiparticles is a significant, perhaps even the dominant, mechanism in suppressing the superfluid density of these two cuprate superconductors. It is interesting that electron-electron interactions appear to be such as to weaken this process.

It is perhaps worth noticing that although the density of superfluid at $T=0$ [$\propto \lambda^{-2}(0)$] is 1.7 times higher in YBCO, the normal fluid density grows at exactly the same rate in both compounds at low temperature. This would naively sug-

gest that T_c should be much higher in YBCO, while it is in fact not very different (5% higher). This is because $\rho_s(T)$ acquires a much stronger downward curvature near T_c in YBCO. Therefore, a major difference must develop at higher temperatures. Part of the answer must come from the very different curvature of the gap function away from the nodes, at high energies, as parametrized by $\mu = S/\Delta_{\max}$ being much larger in YBCO (see Table I).

Specific heat

We complete our quantitative analysis by looking at the electronic specific heat, which is simply derived from Eq. (2), in the clean limit:

$$C_{el}(T) = \frac{18\zeta(3)}{\pi} \frac{k_B^3}{\hbar^2} \frac{n}{d} \left(\frac{1}{v_F v_2} \right) T^2, \quad (10)$$

where $\zeta(3) \approx 1.20$. Extracting this electronic contribution from the total specific heat has been a controversial exercise, since the data can be fitted equally well without a T^2 term. In YBCO at optimal doping, the value quoted in the literature is $0.1 \text{ mJ K}^{-3} \text{ mol}^{-1}$,^{24,25} albeit with a $\pm 60\%$ uncertainty. Using $v_F = 2.5 \times 10^7 \text{ cm/s}$ and $v_2 = 1.8 \times 10^6 \text{ cm/s}$, Eq. (10) gives $0.065 \text{ mJ K}^{-3} \text{ mol}^{-1}$ —a value within the experimental uncertainty.

An alternative approach is to extract v_2 from the field dependence of the specific heat. The Doppler shift of quasiparticle states near the nodes in the presence of the superfluid flow around vortices leads to an increase in the specific heat proportional to \sqrt{H} .²⁶ In terms of the nodal spectrum, the magnitude of the effect is related only to the slope of the gap at the node v_2 . In the clean limit the electronic specific heat of CuO_2 planes (per unit volume), calculated by averaging the effect of the Doppler shift over a single vortex-lattice unit cell, is given by²⁷

$$\frac{C_{el}}{T} = \frac{4k_B^2}{3\hbar} \sqrt{\frac{\pi n}{\Phi_0 d}} \left(\frac{a}{v_2} \right) \sqrt{H}, \quad (11)$$

where Φ_0 is the flux quantum and a is a vortex-lattice parameter of order unity. Such a \sqrt{H} dependence has been seen in measurements on YBCO (Refs. 24 and 25) and the coefficient, of magnitude $0.9 \text{ mJ K}^{-2} \text{ mol}^{-1} \text{ T}^{-1/2}$, determined with greater accuracy ($\pm 10\%$) than the corresponding zero-field T^2 term. Using $v_2 = 1.8 \times 10^6 \text{ cm/s}$ and $a = 1$ in Eq. (11) gives $0.6 \text{ mJ K}^{-2} \text{ mol}^{-1} \text{ T}^{-1/2}$.

It is clear that both aspects of the specific heat data for YBCO are in reasonable quantitative agreement with our

thermal conductivity result for v_F/v_2 and the value of v_F from ARPES. In a refined treatment, one would take into account the contribution of CuO chains to the density of states (and hence to the specific heat). In this respect, it is interesting that Junod and co-workers extract T^2 and \sqrt{H} coefficients for an *overdoped* crystal of YBCO that are somewhat larger:²⁸ $0.20 \pm 0.05 \text{ mJ K}^{-3} \text{ mol}^{-1}$ and $1.3 \pm 0.1 \text{ mJ K}^{-2} \text{ mol}^{-1} \text{ T}^{-1/2}$. It is not unreasonable to attribute this increase to a larger chain density of states, such as would explain the decreasing *a-b* anisotropy in the linear temperature drop of the superfluid density observed in crystals of YBCO as one moves from overdoped to underdoped.²⁰

In conclusion, we have provided a quantitative analysis of low-temperature data for the cuprate superconductors YBCO and BSCCO at optimal doping that compared results from our thermal conductivity measurements with existing results from ARPES, microwave penetration depth, and specific heat. Within a Fermi-liquid model of *d*-wave quasiparticle excitations with interactions, we find all data consistent with a single set of parameters. The Fermi velocity and the Fermi-liquid renormalization of charge currents are found to be roughly the same in both compounds (as is T_c), with $v_F \approx 2.5 \times 10^7 \text{ cm/s}$ and $\alpha^2 \approx 0.4-0.5$, whereas the slope of the gap at the node is about 1.5 times steeper in YBCO.

In particular, the thermal excitation of quasiparticles emerges as a sufficient mechanism for suppressing the superfluid, and there is no clear evidence for a significant contribution from phase fluctuations at low temperature, at least at optimal doping. The success of a Fermi-liquid description for the low-temperature properties should not be taken to mean that the normal state of cuprates is a Fermi liquid. Nor should it be viewed as supporting a BCS theory of the superconducting state, given the fact that, for example, the thermal excitation of quasiparticles does not have the expected impact on the gap itself,³ which remains undiminished up to high temperatures.¹⁶

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